JOHN ADAMSKI
12/13/14

8. $2 + 0.5 + 0.125 + \ldots = \sum_{n=1}^{\infty} 2\left(\frac{1}{4}\right)^{n-1}$

$= \frac{2}{1 - \frac{1}{4}} = \boxed{\frac{8}{3}}$ (CONVERGES)

9. $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n} = \sum_{n=1}^{\infty} \frac{1}{4}\left(-\frac{3}{4}\right)^{n-1} = \frac{\frac{1}{4}}{1 + \frac{3}{4}} = \boxed{\frac{1}{7}}$

11. $\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}} = \sum_{n=1}^{\infty} \frac{1}{3}\left(\frac{\pi}{3}\right)^{n-1}$

$\boxed{\text{DIVERGES}}$ SINCE $\left|\frac{\pi}{3}\right| > 1$.

16. $\sum_{n=1}^{\infty} \frac{1 + 3^n}{2^n} = \sum_{n=1}^{\infty} \frac{1}{2^n} + \underbrace{\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n}$

$\boxed{\text{DIVERGES}}$ BECAUSE $\left|\frac{3}{2}\right| > 1$.

(& SO THE WHOLE SERIES DIVERGES TOO)

21. $\sum_{n=1}^{\infty} \arctan(n)$ $\boxed{\text{DIVERGES}}$ BECAUSE $\lim_{n \to \infty} \arctan(n) \neq 0$

$= \frac{\pi}{2}$

24. $\underbrace{\frac{1}{3} + \frac{2}{9}} + \underbrace{\frac{1}{27} + \frac{2}{81}} + \underbrace{\frac{1}{243} + \frac{2}{729}} + \cdots$

$$= \frac{5}{9} + \frac{5}{81} + \frac{5}{729} + \cdots$$

$$= \sum_{n=1}^{\infty} \frac{5}{9}\left(\frac{1}{9}\right)^{n-1} = \frac{\frac{5}{9}}{1 - \frac{1}{9}} = \boxed{\frac{5}{8}}$$

35. $\sum_{n=1}^{\infty} (-5)^n x^n = \sum_{n=1}^{\infty} (-5x)\underbrace{(-5x)}^{n-1}$

CONVERGES IF $|-5x| < 1$

$|x| < \frac{1}{5}$

i.e. $\boxed{-\frac{1}{5} < x < \frac{1}{5}}$

37. $\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n} = \sum_{n=0}^{\infty} \left(\frac{x-2}{3}\right)^n$

CONVERGES IF $\left|\frac{x-2}{3}\right| < 1$

$|x-2| < 3$

i.e. $\boxed{-1 < x < 5}$