§8.4 OTHER CONVERGENCE TESTS

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3. $\frac{4}{7} - \frac{4}{8} + \frac{4}{9} - \frac{4}{10} + \frac{4}{11} - \ldots$

$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1} 4}{n+6}$ $\boxed{\text{CONVERGES}}$ BECAUSE $\lim_{n\to\infty} \frac{4}{n+6} = 0$

BY ALTERNATING SERIES TEST

7. $\sum_{n=1}^{\infty} (-1)^n \underbrace{\frac{3n-1}{2n+1}}$

$\lim_{n\to\infty} \frac{3n-1}{2n+1} = \frac{3}{2} \neq 0$

$\therefore$ THE SERIES $\boxed{\text{DIVERGES}}$ BY ALT. SERIES TEST.

8. $\sum_{n=1}^{\infty} (-1)^n \underbrace{\cos\left(\frac{\pi}{n}\right)}$

$\boxed{\text{DIVERGES}}$ BY ALT. SERIES TEST. $\left(\lim_{n\to\infty} \cos\frac{\pi}{n} = 1 \neq 0\right)$

19. $\sum_{n=1}^{\infty} \frac{n}{5^n} = \sum_{n=1}^{\infty} \left| \frac{n}{5^n} \right|$ SO EITHER THE SERIES IS ABSOLUTELY CONVERGENT OR IT IS DIVERGENT.

RATIO TEST: $\frac{a_{n+1}}{a_n} = \frac{\frac{n+1}{5^{n+1}}}{\frac{n}{5^n}} = \frac{n+1}{5^{n+1}} \cdot \frac{5^n}{n} = \frac{n+1}{5n}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{1}{5}$. $\therefore$ $\boxed{\text{SERIES CONVERGES ABSOLUTELY BY RATIO TEST}}$

27. $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{(n+1)\,4^{2n+1}}$

RATIO TEST : $\displaystyle\left|\frac{a_{n+1}}{a_n}\right| = \frac{10^{n+1}}{(n+2)\,4^{2n+3}} \cdot \frac{(n+1)\,4^{2n+1}}{10^n}$

$\displaystyle = \frac{n+1}{n+2} \cdot \frac{10}{16} \longrightarrow \frac{5}{8} \text{ AS } n \to \infty$

< 1 $\therefore$ $\boxed{\text{CONVERGES ABSOLUTELY}}$

29. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi/3)}{n!}$

RATIO TEST : $\displaystyle\left|\frac{a_{n+1}}{a_n}\right| = \frac{\cos((n+1)\pi/3)}{(n+1)!} \cdot \frac{n!}{\cos(n\pi/3)}$

$\displaystyle = \frac{\cos((n+1)\pi/3)}{(n+1)\cos(n\pi/3)} \longrightarrow 0 \text{ AS } n \to \infty$

$\therefore$ $\boxed{\text{CONVERGES ABSOLUTELY}}$

33. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$

ROOT TEST : $\displaystyle\sqrt[n]{\left|\left(\frac{n^2+1}{2n^2+1}\right)^n\right|} = \frac{n^2+1}{2n^2+1} \longrightarrow \frac{1}{2} \text{ AS } n \to \infty$

< 1

$\therefore$ $\boxed{\text{ABSOLUTELY CONVERGENT BY ROOT TEST}}$

35. $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{n^2}$

ROOT TEST : $\left| \left(1+\frac{1}{n}\right)^{n^2} \right|^{\frac{1}{n}} = \left(1+\frac{1}{n}\right)^{n} \longrightarrow e$ AS $n \to \infty$

> 1

$\therefore$ $\boxed{\text{DIVERGENT}}$ BY ROOT TEST

36. $\sum_{n=1}^{\infty} \frac{2^{n^2}}{n!}$

RATIO TEST: $\frac{2^{(n+1)^2}}{(n+1)!} \cdot \frac{n!}{2^{n^2}} = \frac{2^{2n+1}}{n+1} \longrightarrow \infty$ AS $n \to \infty$

$\therefore$ $\boxed{\text{DIVERGENT}}$ BY RATIO TEST.