Name: *Answer Key* **10/4/2018**

Math 20300-ST Calculus III **Quiz 2**

Please show all work and **box your final answers**. If you need more room, you may use the backs of the pages. Calculators are not allowed. Good luck!

1. (8 points) Find parametric equations for the line *segment* from $(3, -2, 1)$ to $(-4, 0, -1)$.

LINE SEGMENT FROM A TO B : $\vec{r}(t) = (1-t)\vec{A} + t\vec{B}$, $0 \le t \le 1$

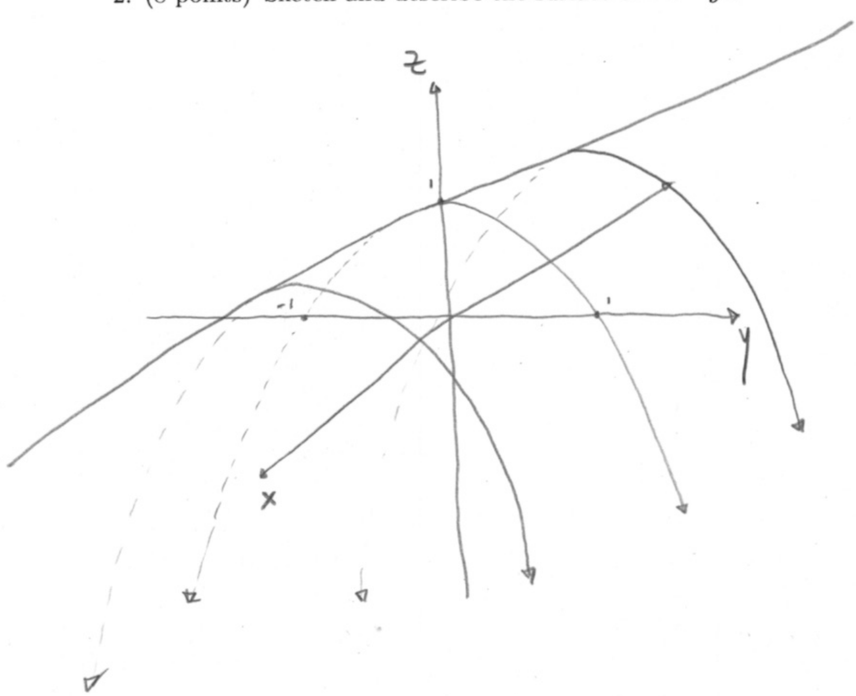
$\therefore \vec{r}(t) = (1-t)\langle 3, -2, 1\rangle + t\langle -4, 0, -1\rangle$

$= \langle (1-t)3 - 4t, -(1-t)2, (1-t) - t\rangle$

$= \langle 3 - 7t, -2 + 2t, 1 - 2t\rangle$

$$\therefore \boxed{\begin{array}{l} x(t) = 3 - 7t \\ y(t) = -2 + 2t \quad , \quad 0 \le t \le 1 \\ z(t) = 1 - 2t \end{array}}$$

2. (8 points) Sketch and describe the surface $z = 1 - y^2$.



$\boxed{\text{PARABOLIC (RIGHT) CYLINDER}}$

3. (8 points) Find an equation for the plane that passes through the points $(0,-2,5)$ and $(-1,3,1)$ and is perpendicular to the plane $2z = 5x + 4y$.

THE PLANE CONTAINS THE POINT $(0,-2,5)$ ✓

SINCE THE PLANE CONTAINS THE VECTOR $\langle -1-0,\ 3-(-2),\ 1-5\rangle$

$= \langle -1, 5, -4\rangle$, ITS NORMAL VECTOR $\vec{n}$ MUST BE $\perp$

TO THIS VECTOR.

SINCE THE PLANE IS $\perp$ TO THE PLANE $5x + 4y - 2z = 0$,

ITS NORMAL VECTOR $\vec{n}$ MUST BE $\perp$ TO THE NORMAL VECTOR

$\langle 5, 4, -2\rangle$.

$\therefore$ $\vec{n}$ MUST BE $\parallel$ TO $\langle -1, 5, -4\rangle \times \langle 5, 4, -2\rangle$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 5 & -4 \\ 5 & 4 & -2 \end{vmatrix} = \langle 6, -22, -29\rangle$$

$\therefore$ EQ OF PLANE : $6x - 22y - 29z = d$

TO FIND d, WE PLUG IN $(0,-2,5)$: $6(0) - 22(-2) - 29(5)$

$= 44 - 145 = -101$

$$\boxed{6x - 22y - 29z = -101}$$

4. (8 points) Consider the space curve defined by the following vector-valued function.

$$\vec{r}(t) = \langle \sqrt{t^2+3},\ \ln(t^2+3),\ t \rangle$$

Give a vector-valued function *and* parametric equations for the tangent line to the space curve $\vec{r}(t)$ at the point $(2, \ln 4, 1)$.

NOTE THAT $\vec{r}(1) = \langle 2, \ln 4, 1 \rangle$

TANGENT LINE : $\ell(t) = \vec{r}(1) + t\,\vec{r}\,'(1)$

$$\vec{r}\,'(t) = \left\langle \frac{\cancel{2}t}{\cancel{2}\sqrt{t^2+3}},\ \frac{2t}{t^2+3},\ 1 \right\rangle$$

$$\vec{r}\,'(1) = \left\langle \frac{1}{2}, \frac{1}{2}, 1 \right\rangle$$

$\therefore\ \ell(t) = \langle 2, \ln 4, 1 \rangle + t \left\langle \frac{1}{2}, \frac{1}{2}, 1 \right\rangle$

OR $\ell(t) = \left\langle 2 + \frac{1}{2}t,\ \ln 4 + \frac{1}{2}t,\ 1 + t \right\rangle$ (VECTOR)

OR $x = 2 + \frac{1}{2}t$

$y = \ln 4 + \frac{1}{2}t$ (PARAMETRIC)

$z = 1 + t$

5. (4 points) State a single MatLab command that will assign to the variable x the vector

$$[7, 17, 27, \ldots, 107]$$

(a) using the `colon` : : operator;

X = [7 : 10 : 107]

(b) using the `linspace` command.

X = LINSPACE (7 , 107 , 11)

6. (4 points) If $x = [1, 2, 3, 4]$, what MatLab command will produce the output $[1^3, 2^3, 3^3, 4^3]$?

X.^3