MATH 212 FINAL EXAM DRAFT A

SOLUTIONS BY JOHN ADAMSKI

1. EVALUATE THE FOLLOWING INTEGRALS.

(a) $\int_1^e x^4 \ln x \, dx$ INTEGRATION BY PARTS.

Let $u = \ln x$ $\quad v = \frac{1}{5}x^5$

$du = \frac{1}{x}dx$ $\quad dv = x^4 dx$

$$\int_1^e u\,dv = uv\Big|_1^e - \int_1^e v\,du = \frac{1}{5}x^5 \ln x \Big|_1^e - \frac{1}{5}\int_1^e x^4 dx$$

$$= \frac{1}{5}\left[e^5 - 0\right] - \frac{1}{25}x^5\Big|_1^e = \boxed{\frac{1}{5}e^5 - \frac{1}{25}\left[e^5 - 1\right]}$$

(b) $\int \cos^3(2t)\sin^5(2t)\,dt = \int (1-\sin^2(2t))\sin^5(2t)\cos(2t)\,dt$

Let $u = \sin(2t)$ $\quad = \frac{1}{2}\int (1-u^2)u^5\,du$

$du = 2\cos(2t)dt$

$$= \frac{1}{2}\int u^5 - u^7\,du$$

$$= \boxed{\frac{1}{2}\left[\frac{1}{6}u^6 - \frac{1}{8}u^8\right] + C}$$

(c) $\int_0^{\sqrt{2}/4} \frac{2\,dt}{\sqrt{1-4t^2}}$ Let $u = 2t$ $\longrightarrow$ $\int_{u=0}^{u=\sqrt{2}/4}$ $\longrightarrow$ $\int_{t=2(0)}^{t=2\left(\sqrt{2}/4\right)}$

$du = 2\,dt$

$$\rightsquigarrow \int_0^{\sqrt{2}/2} \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1}u \Big|_0^{\sqrt{2}/2} = \sin^{-1}\frac{\sqrt{2}}{2} - \sin^{-1}0$$

$$= \frac{\pi}{4} - 0 = \boxed{\frac{\pi}{4}}$$

(d) $\int \frac{e^t}{e^{2t} + 3e^t + 2}\,dt$ Let $u = e^t$

$du = e^t\,dt$

$$\rightsquigarrow \int \frac{1}{u^2 + 3u + 2}\,du = \int \frac{1}{(u+2)(u+1)}\,du$$

PARTIAL FRACTIONS: $\frac{1}{(u+2)(u+1)} = \frac{A}{u+2} + \frac{B}{u+1}$

$$\Rightarrow \quad 1 = A(u+1) + B(u+2)$$

$u = -2 \Rightarrow 1 = -A \Rightarrow A = -1$

$u = -1 \Rightarrow 1 = B \Rightarrow B = 1$