§10.2 INFINITE SERIES

GIVEN A SEQUENCE $a_1, a_2, a_3, \ldots, a_n, \ldots$

THERE IS A RELATED SERIES

$$s_1 = a_1$$
$$s_2 = a_1 + a_2$$
$$s_3 = a_1 + a_2 + a_3$$
$$\vdots$$
$$s_n = a_1 + a_2 + \ldots + a_n \quad (n^{\text{TH}} \text{ PARTIAL SUM})$$
$$\vdots$$

A SEQUENCE OF PARTIAL SUMS

$$S_n = \sum_{j=1}^{n} a_j$$

GIVEN A SERIES, THERE IS A RELATED SEQUENCE

$$a_1 = s_1$$
$$a_n = s_n - s_{n-1}$$

THEN $S_n = a_n + S_{n-1} = a_n + a_{n-1} + S_{n-2} = \ldots a_n + a_{n-1} + \ldots + a_2 + a_1$.

SEQUENCE $a_n = \left(\frac{1}{2}\right)^n$ SERIES $S_n = \sum_{i=1}^{n} \left(\frac{1}{2}\right)^i$

Def: IF THE SEQUENCE OF PARTIAL SUMS $S_n = \sum_{j=1}^{n} a_j$ CONVERGES TO A LIMIT L, WE SAY THE SERIES $S = \sum_{j=1}^{\infty} a_j$ CONVERGES AND THAT ITS SUM IS L.

IF THE SEQ OF PARTIAL SUMS DOES NOT CONVERGE, THEN THE SERIES DIVERGES.

Geometric Series:

$$a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$$

$$a \quad a_2 \quad a_3 \quad a_4 \quad a_n$$

e.g. $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \frac{80}{81} - \dots$

$a = 5$ $\quad \times r \quad \times -\frac{2}{3}$

Converges to

$$\frac{5}{1-(-\frac{2}{3})} = 3$$

$$\begin{aligned} S_n &= a + ar + ar^2 + \dots + ar^{n-1} \\ - \quad rS_n &= \quad\;\; ar + ar^2 + \dots + ar^{n-1} + ar^n \\ \hline (1-r)S_n &= a - ar^n \end{aligned}$$

$$S_n = \frac{a(1-r^n)}{1-r}, \quad r \neq 1$$

Note: if $|r| < 1$ then $\lim_{n\to\infty} r^n = 0$.

$\therefore$ $\boxed{\text{For Geo. Series } a + ar + ar^2 + \dots \text{ with } |r|<1, \quad \lim_{n\to\infty} S_n = \frac{a}{1-r} \quad \text{i.e.} \quad \sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}.}$

(If $|r| \geq 1$ then series diverges)