

ex.

$$\int_0^{\pi/4} 3^{\cos(2t)} \sin(2t) dt$$

$$y = a^x = e^{(\ln a)x}$$

$$y' = e^{(\ln a)x} \ln a = a^x \ln a$$

$$u = \cos(2t)$$

$$du = (-2 \sin(2t)) dt$$

$$-\frac{1}{2} du = \sin(2t) dt$$

$$\frac{d}{dx} a^x = a^x \ln a \quad \left\{ \int a^x dx = \frac{1}{\ln a} a^x + C \right.$$

SWITCH BOUNDS
+
NEGATE

$$\int_0^{\pi/4} 3^{\cos(2t)} \sin(2t) dt = -\frac{1}{2} \int_1^0 3^u du$$

$$\frac{1}{2} \int_0^1 3^u du = \frac{1}{2} \left[\frac{1}{\ln 3} 3^u \right]_0^1$$

$$= \frac{1}{2 \ln 3} (3^1 - 3^0) = \frac{2}{2 \ln 3} = \boxed{\frac{1}{\ln 3}}$$

CHANGE VARIABLE \Rightarrow CHANGE BOUNDS

$$t = \pi/4 \longrightarrow u = \cos(2 \cdot \frac{\pi}{4}) = 0$$

$$\int \longrightarrow \int$$

$$t = 0 \longrightarrow u = \cos(2 \cdot 0) = 1$$

§ 8.1 USING BASIC INTEGRATION FORMULAS

0. u-SUBSTITUTION

1. COMPLETE THE SQ.

2. TRIG IDENTITIES

PYTHAGOREAN IDENTITY

$$\frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 = \sec^2 \theta - \tan^2 \theta^*$$

ex. $\int \frac{1}{\sec \theta + \tan \theta} d\theta$

$$= \int \frac{(\sec \theta - \tan \theta)^2}{(\sec \theta + \tan \theta)(\sec \theta - \tan \theta)} d\theta \quad \begin{matrix} (a+b)(a-b) \\ a^2 - b^2 \end{matrix}$$

$$= \int \frac{\sec \theta - \tan \theta}{\sec^2 \theta - \tan^2 \theta^*} d\theta = \int \sec \theta - \tan \theta d\theta$$

$$= \int \sec \theta d\theta - \int \tan \theta d\theta$$

(1) (2)

(2) $\int \tan \theta d\theta$

$$= \int \frac{\sin \theta}{\cos \theta} d\theta \quad \begin{matrix} u = \cos \theta \\ du = -\sin \theta d\theta \\ -du = \sin \theta d\theta \end{matrix}$$

(1) $\int \sec \theta d\theta = \int \sec \theta \cdot \frac{\tan \theta + \sec \theta}{\tan \theta + \sec \theta} d\theta \rightsquigarrow \int \frac{1}{u} du = -\ln|u| = -\ln|\cos \theta| + C$

$$= \int \frac{\sec \theta \tan \theta + \sec^2 \theta}{\tan \theta + \sec \theta} d\theta$$

let $u = \tan \theta + \sec \theta$
 $du = (\sec \theta \tan \theta + \sec^2 \theta) d\theta$

$$\rightsquigarrow \int \frac{1}{u} du = \ln|u| + C \rightsquigarrow \ln|\tan \theta + \sec \theta| + C$$

(1)

$$\textcircled{2} \dots \textcircled{1} \rightarrow \ln |\cos \theta| + c = \ln (|\cos \theta|^{\ominus}) + c$$

$$= \ln |\sec \theta| + c$$

$$\therefore \textcircled{1} - \textcircled{2} = \ln |\tan \theta + \sec \theta| - \ln |\sec \theta| + c$$

3. MULTIPLY TOP & BOTTOM OF FRACTIONS BY RADICAL CONJUGATES

EXPRESSION	CONJUGATE	"COUSIN"	TRIG IDENTITIES!
$a + b$	$a - b$	MULTIPLY: $(a+b)(a-b) = a^2 - b^2$	
$a + ib$	$a - ib$	"COMPLEX CONJUGATE"	
$a + \sqrt{b}$	$a - \sqrt{b}$	"RADICAL CONJUGATE"	

$$(a + \sqrt{b})(a - \sqrt{b}) = a^2 - a\sqrt{b} + a\sqrt{b} - b$$

$$= a^2 - b \quad (\text{NO RADICAL!})$$

ex. $\int \sqrt{1 - \cos \theta} \, d\theta = \int \sqrt{\frac{(1 - \cos \theta)(1 + \cos \theta)}{1 + \cos \theta}} \, d\theta$

$$= \int \sqrt{\frac{1 - \cos^2 \theta}{1 + \cos \theta}} \, d\theta = \int \sqrt{\frac{\sin^2 \theta}{1 + \cos \theta}} \, d\theta$$

$$= \int \frac{\sin \theta}{\sqrt{1 + \cos \theta}} \, d\theta$$

LET $u = 1 + \cos \theta$
 $-du = \sin \theta \, d\theta$

$$\rightarrow - \int \frac{1}{\sqrt{u}} du = - \int u^{-1/2} du = -2u^{1/2} + C$$

$$\rightarrow \boxed{2(1 + \cos \theta)^{1/2} + C}$$

4. POLYNOMIAL LONG DIVISION,

"IMPROPER"

DEGREE OF NUM.

BIGGER THAN (\geq)

DEGREE OF DENOM.

$$\frac{3}{7}$$

"PROPER"

$$3 < 7$$

ex.

$$\int \frac{8x^2 + 10x + 3}{4x + 5} dx$$

$$\begin{array}{r} 2x \\ 4x + 5 \overline{) 8x^2 + 10x + 3} \\ \underline{-(8x^2 + 10x)} \\ 3 \end{array}$$

$$7 \overline{) 43} \\ \underline{-42} \\ 1$$

$$\frac{43}{7} = 6 + \frac{1}{7}$$

1 REMAINDER

$$\frac{43}{7} \text{ "IMPROPER" } \rightarrow 6 \frac{1}{7}$$

$$\int 2x + \frac{3}{4x + 5} dx$$

$$= x^2 + C + \int \frac{3}{4x + 5} dx$$

$$\text{let } u = 4x + 5$$

$$du = 4 dx$$

$$\frac{1}{4} du = dx$$

$$= x^2 + C + \frac{3}{4} \int \frac{1}{u} du = x^2 + \frac{3}{4} \ln |u| + C$$

$$\rightarrow \boxed{x^2 + \frac{3}{4} \ln |4x + 5| + C}$$

SUMMARY : $\int \frac{\text{DEGREE } n \text{ POLYNOMIAL}}{\text{DEGREE } m \text{ POLYNOMIAL}} dx$

USE POLYNOMIAL LONG DIVISION WHEN $n \geq m$.

(DEGREE = HIGHEST EXPONENT)

ex. $\int \frac{x^4}{x^2 + 1} dx$

vs.

$\int \frac{x^2 + 1}{x^4} dx$

LONG DIV.

NOT LONG DIV.

$$\begin{array}{r} \textcircled{x^2 + 1} \overline{) \begin{array}{r} x^4 \\ - (x^4 + x^2) \\ \hline -x^2 \\ - (-x^2 - 1) \\ \hline 1 \end{array}} \end{array}$$

① REMAINDER

$$\int \frac{x^2}{x^4} + \frac{1}{x^4} dx = \int x^{-2} + x^{-4} dx$$

$$= \boxed{-x^{-1} - \frac{1}{3}x^{-3} + C}$$

$$= \int x^2 - 1 + \frac{1}{x^2 + 1} dx$$

$$= \boxed{\frac{1}{3}x^3 - x + \tan^{-1}x + C}$$

