(1 1 1 2
7.1, 7.3, 8.1, 8.2)

QUIZ # 1 TOMORROW (5 QUIZZES TOP 3 QUIZZES WILL COUNT)

8:30 → ZOOM

8:40 → PEARSON QUIZ 1 APPEARS

30 MIN TO COMPLETE 5 QUESTIONS.

9:15 → RESUME CLASS ON ZOOM

I'LL BE IN ZOOM MEETING DURING QUIZ.

ex. §8.1

$$\int \frac{1}{64e^{-5x}+e^{5x}}\,dx = \int \frac{1}{64e^{-5x}+e^{5x}} \cdot \frac{e^{5x}}{e^{5x}}\,dx$$

$$= \int \frac{e^{5x}}{64+(e^{5x})^2}\,dx$$

$u = e^{5x}$

$du = 5e^{5x}\,dx \rightarrow \frac{1}{5}du = e^{5x}\,dx$

$$= \frac{1}{5}\int \frac{1}{64+u^2}\,du = \frac{1}{5}\left(\frac{1}{8}\tan^{-1}\left(\frac{u}{8}\right)\right) + C$$

$$\rightarrow \boxed{\frac{1}{40}\tan^{-1}\left(\frac{e^{5x}}{8}\right) + C}$$

ex. §8.2

$$\int \underbrace{\sin(2x)}_{u}\underbrace{\cos(7x)\,dx}_{dv}$$

$u = \sin(2x)$ $\quad v = \frac{1}{7}\sin(7x)$

$du = 2\cos(2x)\,dx$ $\quad dv = \cos(7x)\,dx$

$$= uv - \int v\,du = \frac{1}{7}\sin(2x)\sin(7x) - \frac{2}{7}\boxed{\int \sin(7x)\cos(2x)\,dx}$$

$u = \cos(2x)$ $\quad v = -\frac{1}{7}\cos(7x)$

$du = -2\sin(2x)\,dx$ $\quad dv = \sin(7x)\,dx$

$$\boxed{\int \sin(2x)\cos(7x)\,dx} = \frac{1}{7}\sin(2x)\sin(7x) - \frac{2}{7}\left[-\frac{1}{7}\cos(2x)\cos(7x) - \frac{2}{7}\boxed{\int \sin(2x)\cos(7x)\,dx}\right]$$

$$\frac{49}{49}\int \sin(2x)\cos(7x)\,dx = \frac{1}{7}\sin(2x)\sin(7x) + \frac{2}{49}\cos(2x)\cos(7x)$$

$$+\frac{4}{49}\int \cancel{\sin(2x)\cos(7x)\,dx}$$

$$-\frac{4}{49}\int \sin(2x)\cos(7x)\,dx \qquad -\frac{4}{49}\int \cancel{\sin(2x)\cos(7x)\,dx}$$

$$\frac{49}{45}\left(\frac{45}{49}\int \cdots\cdots\, dx\right) = \left(\cdots\cdots\right)\frac{49}{45}$$

$$\int \sin(2x)\cos(7x)\,dx = \boxed{\frac{49}{45}\left(\frac{1}{7}\sin(2x)\sin(7x) + \frac{2}{49}\cos(2x)\cos(7x)\right) + C}$$

§8.3 TRIGONOMETRIC INTEGRALS

$$\boxed{\int \sin^m x \cos^n x \, dx}$$

•) IF m ODD : $\displaystyle\int \overbrace{\underbrace{\sin^{m-1} x}_{} \cos^n x}^{\text{ALL COSINE}} \; \underbrace{\sin x \, dx}_{}$

EVEN # OF SIN'S : $\sin^2 x = 1 - \cos^2 x$

$u = \cos x \qquad -du = \sin x \, dx$

•) IF n ODD : $\displaystyle\int \overbrace{\sin^m x \underbrace{\cos^{n-1} x}_{}}^{\text{ALL SIN'S}} \; \underbrace{\cos x \, dx}_{du}$

REWRITE AS EXPRESSION OF SIN X

$\cos^2 x = 1 - \sin^2 x$

$u = \sin x \qquad du = \cos x \, dx$

•) IF BOTH m, n EVEN : TRIG IDENTITIES.

$$\left.\begin{aligned} \cos^2 x &= \tfrac{1}{2}(1 + \cos 2x) \\ \sin^2 x &= \tfrac{1}{2}(1 - \cos 2x) \end{aligned}\right\} \text{REDUCES THE EXPONENT!}$$

ex. $\int \sin^5 x \cos^7 x \, dx$

$\int \sin^4 x \cos^7 x \underbrace{\sin x \, dx}_{du}$

$\sin^4 x = \left(\sin^2 x\right)^2$

$= \left(1 - \cos^2 x\right)^2$

$\int \overbrace{\left(1-\cos^2 x\right)^2 \cos^7 x}^{\text{ALL COS}} \, \sin x \, dx$

$u = \cos x \qquad -du = \sin x \, dx$

$-\int (1-u^2)^2 u^7 \, du$

$-\int (1 - 2u^2 + u^4) u^7 \, du$

$-\int u^7 - 2u^9 + u^{11} \, du$

$-\frac{1}{8} u^8 + \frac{1}{5} u^{10} - \frac{1}{12} u^{12} + C$

$-\frac{1}{8} \cos^8 x + \frac{1}{5} \cos^{10} x - \frac{1}{12} \cos^{12} x + C$

$\int \sin^5 x \cos^6 x \underbrace{\cos x \, dx}_{du}$

$\cos^6 x = \left(\cos^2 x\right)^3$

$= \left(1 - \sin^2 x\right)^3$

$\int \sin^5 x \left(1 - \sin^2 x\right)^3 \cos x \, dx$

$u = \sin x \qquad du = \cos x \, dx$

$\vdots$

ex. $\int \cos^4 x \, dx$

Note: $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$$\cos^4 x = \left[\frac{1}{2}(1 + \cos 2x)\right]^2$$

$$= \frac{1}{4}\left(1 + 2\cos 2x + \cos^2 2x\right)$$

$$= \frac{1}{4}\int 1 + 2\cos 2x + \underbrace{\cos^2 2x}_{\frac{1}{2}(1+\cos(2\cdot 2x))} \, dx$$

$$= \frac{1}{4}\left[x + \sin 2x + \frac{1}{2}\int 1 + \cos 4x \, dx\right]$$

$$= \frac{1}{4}\left[x + \sin 2x + \frac{1}{2}\left(x + \frac{1}{4}\sin 4x\right)\right] + C$$

$$= \frac{1}{4}x + \frac{1}{4}\sin 2x + \frac{1}{8}x + \frac{1}{32}\sin 4x + C$$

$$= \frac{3}{8}x + \frac{1}{8}\sin 2x + \frac{1}{32}\sin 4x + C$$

$$\boxed{\int \tan^m x \sec^n x \, dx}$$

·) IF m IS ODD : $\displaystyle\int \underbrace{\tan^{m-1} x \sec^{n-1} x}_{(\text{ALL SEC})} \tan x \sec x \, dx$

$\tan^2 x = \sec^2 x - 1$

$u = \sec x \qquad du = \sec x \tan x \, dx$

·) IF n IS EVEN : $\displaystyle\int \underbrace{\tan^m x \sec^{n-2} x}_{(\text{ALL TAN})} \underbrace{\sec^2 x \, dx}$

$\sec^2 x = \tan^2 x + 1$

$u = \tan x \qquad du = \sec^2 x \, dx$

·) IF m IS EVEN , n IS ODD : TRIG IDENTITIES ,

* INTEGRATION BY PARTS ,

ex. $\int \tan^2 x \sec^4 x \, dx = \int \underbrace{\tan^2 x \sec^2 x}_{\text{ALL TAN } (= u)} \overbrace{\sec^2 x \, dx}^{du}$

$= \int \tan^2 x (\tan^2 x + 1) \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$= \int u^2 (u^2 + 1) \, du = \int u^4 + u^2 \, du = \frac{1}{5} u^5 + \frac{1}{3} u^3 + C$

$\rightsquigarrow \boxed{\frac{1}{5} \tan^5 x + \frac{1}{3} \tan^3 x + C}$

ex. $\int \tan^3 x \sec^3 x \, dx = \int \underbrace{\tan^2 x \sec^2 x}_{(\text{ALL SEC } x)} \underbrace{\sec x \tan x \, dx}_{du}$

$\frac{d}{dx}$

$= \int (\sec^2 x - 1) \sec^2 x \sec x \tan x \, dx$

$u = \sec x$

$du = \sec x \tan x \, dx$

$\rightsquigarrow \int (u^2 - 1) u^2 \, du = \int u^4 - u^2 \, du = \frac{1}{5} u^5 - \frac{1}{3} u^3 + C$

$\rightsquigarrow \boxed{\frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C}$

$\sec^4 x - \sec^2 x$

$= \tan^3 x \sec^3 x$?

Yes.

ex. $\int \sec^3 x \, dx = \int \sec x \sec^2 x \, dx$

$u = \sec x$ $\quad v = \tan x$

$du = \sec x \tan x \, dx$ $\quad dv = \sec^2 x \, dx$

$$= uv - \int v\,du = \sec x \tan x - \int \underbrace{\tan^2 x}_{} \sec x \, dx$$

$(\sec^2 x - 1) \sec x$

$\sec^3 x - \sec x$

$$\int \sec^3 x \, dx = \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$$

$$2 \int \sec^3 x \, dx = \sec x \tan x + \ln\left|\sec x + \tan x\right|$$

$$\int \sec^3 x \, dx = \boxed{\frac{1}{2}\left(\sec x \tan x + \ln\left|\sec x + \tan x\right| \right) + C}$$

§ 8.4 TRIGONOMETRIC SUBSTITUTIONS

GIVEN $\displaystyle\int_0^{\pi/2} \sqrt{1-\sin^2\theta}\,\cos\theta\, d\theta$

$\theta = \pi/2$ $\qquad$ $x = \sin\left(\frac{\pi}{2}\right) = 1$

$$\int_{\theta=0}^{\theta=\pi/2} \xrightarrow{x = \sin\theta} \int_{x=\sin(0)=0}^{x=\sin(\pi/2)=1}$$

U-SUB.

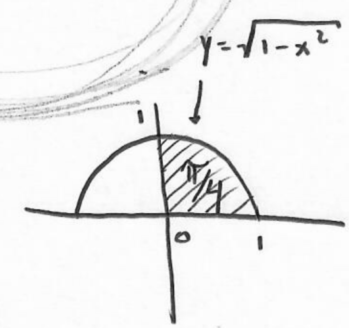
TRIG. SUB

USE X - SUBSTITUTION

$x = \sin\theta$

$dx = \cos\theta\, d\theta$

$$\int_0^1 \sqrt{1-x^2}\, dx$$



BOTH INTEGRALS HAVE THE SAME ANSWER.

IN FACT, THE FIRST INTEGRAL IS EASIER! TRIG IDENTITIES!

$$\int_0^{\pi/2} \sqrt{1-\sin^2\theta}\,\cos\theta\, d\theta = \int_0^{\pi/2} \sqrt{\cos^2\theta}\,\cos\theta\, d\theta$$

$$= \int_0^{\pi/2} |\cos\theta|\cos\theta\, d\theta = \int_0^{\pi/2} \cos^2\theta\, d\theta$$

$0 \le \theta \le \frac{\pi}{2} \Rightarrow \cos\theta \ge 0$

$$= \frac{1}{2}\int_0^{\pi/2} 1 + \cos 2\theta\, d\theta = \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right)\Bigg|_0^{\pi/2}$$

$$= \boxed{\frac{\pi}{4}}$$

3 TRIG SUBSTITUTIONS:

1.

EXPRESSION: $\sqrt{a^2 - x^2}$; $-a \le x \le a$ (ASSUME $a > 0$)

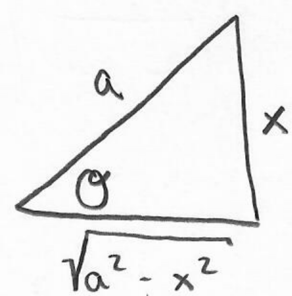
SUBSTITUTION: $x = a\sin\theta$ $\quad -\frac{\pi}{2} \le \theta = \sin^{-1}\frac{x}{a} \le \frac{\pi}{2}$

$dx = a\cos\theta\, d\theta$

SIMPLIFY: $\sqrt{a^2 - x^2} = \sqrt{a^2 - a^2\sin^2\theta} = \sqrt{a^2}\sqrt{1 - \sin^2\theta}$

$= |a||\cos\theta| = a\cos\theta$ (NO $\sqrt{}$!)

NOTE: $x = a\sin\theta \iff \sin\theta = \frac{x}{a}$ $\quad \frac{\text{OPP}}{\text{HYP}}$



$\cos\theta = \frac{\sqrt{a^2 - x^2}}{a}$

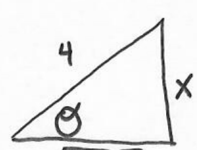
$a\cos\theta = \sqrt{a^2 - x^2}$ ✓

(PYTH. THM.)

ex. $\int \frac{1}{\sqrt{16 - x^2}}\, dx \quad \left(= \sin^{-1}\left(\frac{x}{4}\right) + C \right)$

LET $x = 4\sin\theta$

$dx = 4\cos\theta\, d\theta$



$\sqrt{16 - x^2} = 4\cos\theta$

$\sin\theta = \frac{x}{4} \Rightarrow \theta = \sin^{-1}\frac{x}{4}$

$\rightsquigarrow \int \frac{1}{4\cos\theta} \cdot 4\cos\theta\, d\theta = \int d\theta = \theta = \boxed{\sin^{-1}\frac{x}{4} + C}$

2.

EXPRESSION : $\sqrt{a^2 + x^2}$, $-\infty < x < \infty$

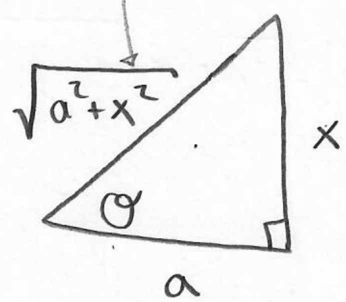
SUBSTITUTION : $x = a \tan \theta \longrightarrow \tan \theta = \dfrac{x}{a}$, $\theta = \tan^{-1} \dfrac{x}{a}$

$\left(-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2} \right)$

$dx = a \sec^2 \theta \, d\theta$

SIMPLIFY : $\sqrt{a^2 + x^2}$

$= a \sec \theta$



$\tan \theta = \dfrac{x}{a}$

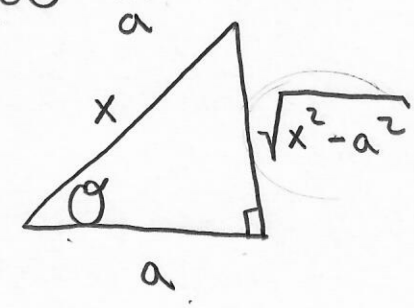
$\sec \theta = \dfrac{\sqrt{a^2 + x^2}}{a}$

3.

EXPRESSION: $\boxed{\sqrt{x^2 - a^2}}$ $\quad \underbrace{x \geq a}_{\text{ASSUME}}$ or $x \leq -a$, $a > 0$

SUBSTITUTION : $x = a \sec \theta \longrightarrow \sec \theta = \dfrac{x}{a}$

$dx = a \sec \theta \tan \theta \, d\theta$



SIMPLIFY: $\sqrt{x^2 - a^2} = a \tan \theta$

$\left(\tan \theta = \dfrac{\sqrt{x^2 - a^2}}{a} \right)$

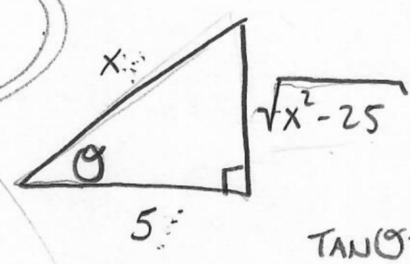
ex. $\displaystyle\int \frac{\sqrt{x^2-25}}{x^3}\,dx$ Let $x = 5\sec\theta$

$dx = 5\sec\theta\tan\theta\,d\theta$

TRIG SUB

U-SUB

$\sec\theta = \dfrac{x}{5}$



$= 5\tan\theta$

$\tan\theta = \dfrac{\sqrt{x^2-25}}{5}$

$$\rightsquigarrow \int \frac{5\tan\theta}{(5\sec\theta)^3}\,\overbrace{5\sec\theta\tan\theta\,d\theta}^{dx}$$

$$= \frac{1}{5}\int \frac{\tan^2\theta}{\sec^2\theta}\,d\theta \qquad \frac{\tan^2\theta}{\sec^2\theta} = \frac{\sin^2\theta}{\cancel{\cos^2\theta}}\cdot\cancel{\cos^2}\theta$$

$$= \frac{1}{5}\int \sin^2\theta\,d\theta = \frac{1}{5}\cdot\frac{1}{2}\int 1-\cos 2\theta\,d\theta$$

$$= \frac{1}{10}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C \qquad \theta = \sec^{-1}\frac{x}{5}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= \frac{1}{10}\left(\theta - \sin\theta\cos\theta\right) = \frac{1}{10}\left(\sec^{-1}\frac{x}{5} - \frac{\sqrt{x^2-25}}{x}\cdot\frac{5}{x}\right) + C$$

$$= \boxed{\frac{1}{10}\sec^{-1}\frac{x}{5} - \frac{\sqrt{x^2-25}}{2x^2} + C}$$