QUIZ TOMORROW §10.7 - 10.

POWER SERIES (WHERE THEY CONVERGE)

TAYLOR / MACLAURIN SERIES

CONVERGENCE OF TAYLOR SERIES

APPLICATIONS.

$f(x) = 7\sin(5x)$ MACLAURIN SERIES. $(x=0)$

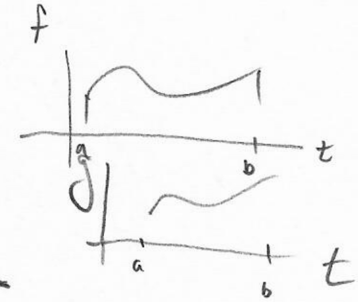
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$7\sin(5x) = 7\left((5x) - \frac{(5x)^3}{3!} + \frac{(5x)^5}{5!} - \frac{(5x)^7}{7!} + \dots \right)$$

$$\sin x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} (-1)^n$$

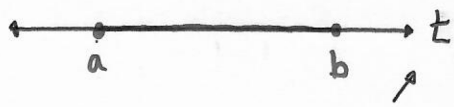
$$7\sin(5x) = \sum 7(-1)^n \frac{(5x)^{2n+1}}{(2n+1)!}$$

§11.1 PARAMETRIC CURVES

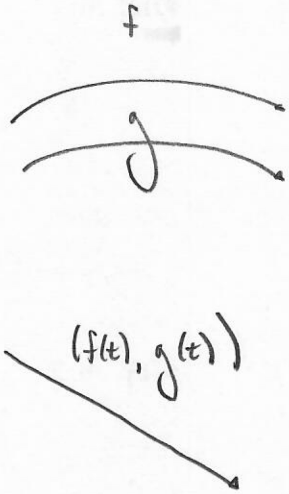


(X)

DOMAIN IS INTERVAL $[a, b]$

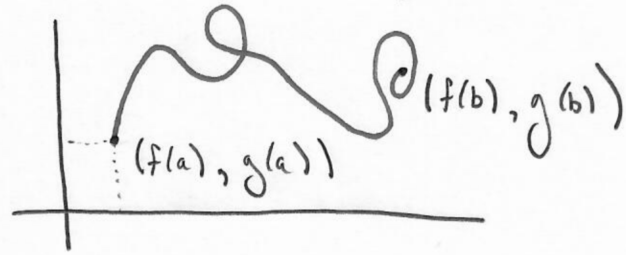


PARAMETER



x, y COORD. OF PARTICLE AT TIME t

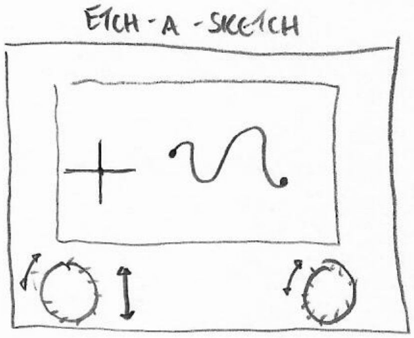
$x = f(t)$, $y = g(t)$



xy - PLANE

(ASSUME f, g ARE DIFFERENTIABLE)

ETCH - A - SKETCH

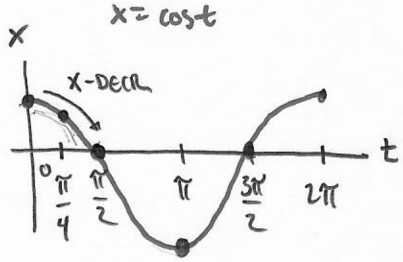


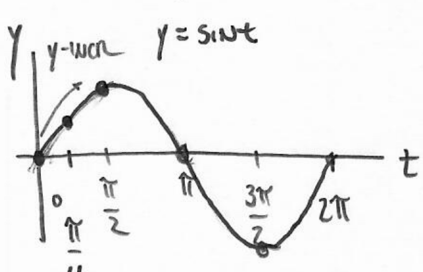
ex. SKETCH THE PARAMETRIC CURVE: $x = \cos t$, $y = \sin t$ $\left(\begin{array}{l} x = f(t) = \cos t \\ y = g(t) = \sin t \end{array} \right)$

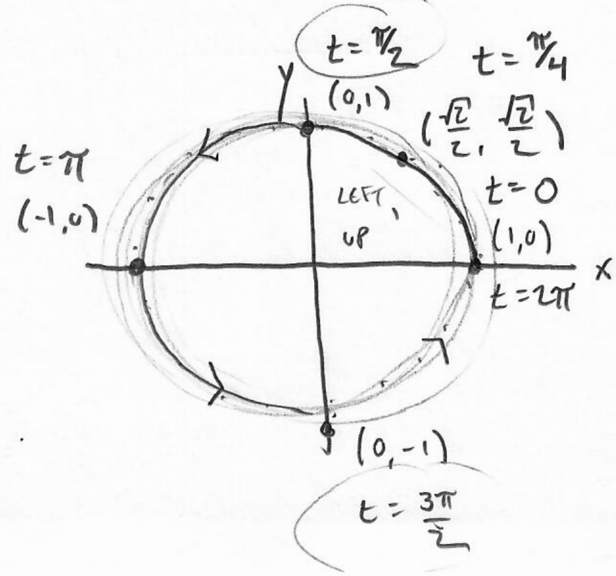
$0 \le t \le 2\pi$

PARAMETER : t

DOMAIN : $[0, 2\pi]$







t	x	y
0	1	0
$\frac{\pi}{2}$	0	1
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
π	-1	0
$\frac{3\pi}{2}$	0	-1
2π	1	0

$x = \cos t$

$y = \sin t$

TO SHOW THAT THE IMAGE OF THIS PARAMETRIC CURVE LIES ON A CIRCLE:

ELIMINATE THE PARAMETER! - SOLVE FOR t, SUBSTITUTE INTO OTHER EQ.

TRIG ID: PYTHAGOREAN IDENTITY:

$$\sin^2 t + \cos^2 t = 1 \quad \text{FOR ALL } t.$$

$$y^2 + x^2 = 1$$

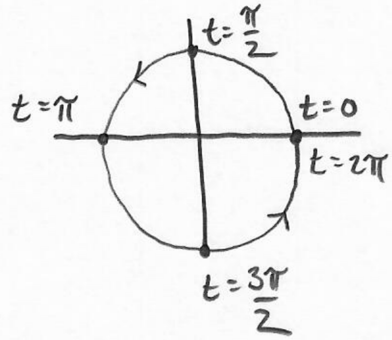
UNIT CIRCLE!

SUMMARY:

$x = \cos t$

$y = \sin t$

$0 \leq t \leq 2\pi$



PARTICLE MOVES $1\times$ AROUND UNIT CIRCLE COUNTER-CLOCKWISE.

NOTE: PARAMETRIC CURVES ARE MORE THAN CURVES.

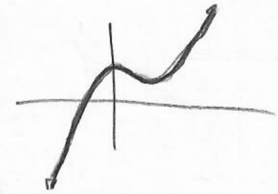
- BEGINNING, END
- DIRECTION

$y = x^3 - 2x + 1$

NO BEGINNING

NO END

NO DIRECTION

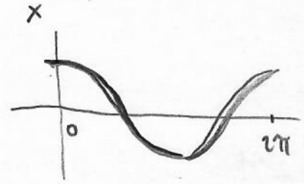


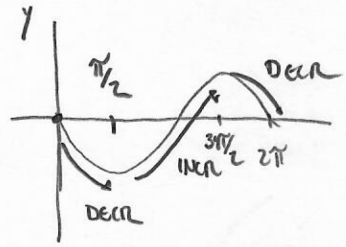
ex.

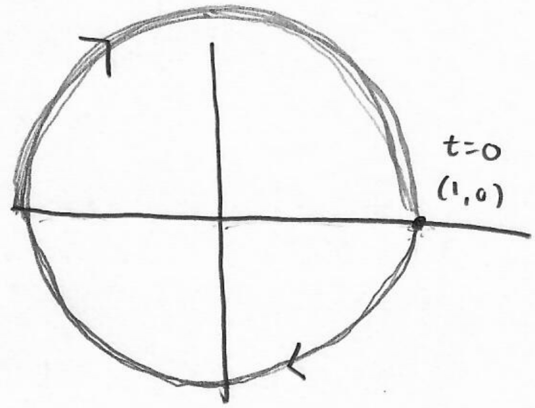
$x = \cos t$

$y = -\sin t$

$0 \le t \le 2\pi$





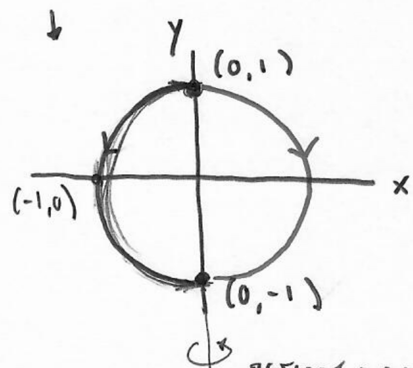


$(x^2 + y^2 = 1)$

PARTICLE MOVES ONCE AROUND UNIT CIRCLE CLOCKWISE.

ex. FIND A PARAMETRIZATION FOR A PARTICLE THAT BEGINS AT $(0,1)$ AND ENDS AT $(0,-1)$ AND MOVES CLOCKWISE AROUND UNIT CIRCLE.

COUNTER CLOCKWISE



REFLECT ACROSS y-AXIS

⇒ NEGATE x-COORD

① $x = \cos t$

$y = \sin t$

$\frac{\pi}{2} \le t \le \frac{3\pi}{2}$

CLOCKWISE:

$x = -\cos t$

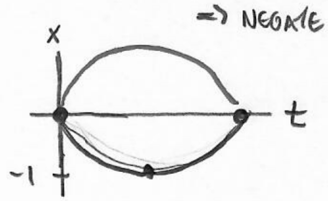
$y = \sin t$

$\frac{\pi}{2} \le t \le \frac{3\pi}{2}$

$x = \sin t$

$y = \cos t$

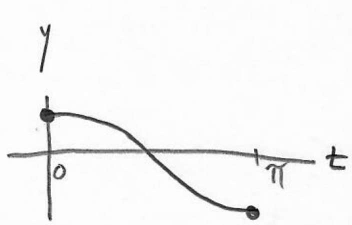
$0 \le t \le \pi$



$x = -\sin t \,, \quad 0 \le t \le \pi$

SAME

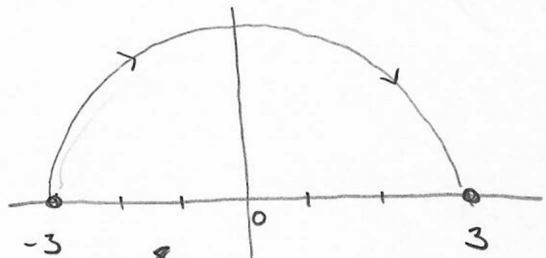
$y = \cos t \,, \quad 0 \le t \le \pi$

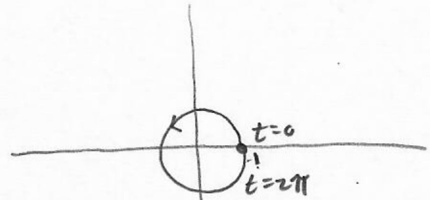


ex. FIND PARAM. FOR PARTICLE STARTING AT $(-3, 0)$

ENDS AT $(3, 0)$, TRAVELLING CLOCKWISE AROUND

A CIRCLE CENTERED AT O.





$x = \cos t$

$y = \sin t$

$0 \le t \le 2\pi$

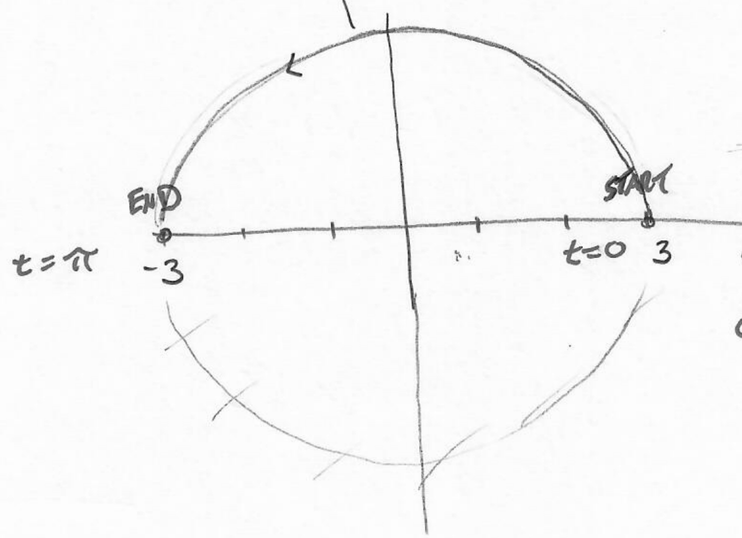
$x = -3\cos t$

$y = 3\sin t$

$0 \le t \le \pi$

NEGATE X-COORD

REFLECT ACROSS Y-AXIS



$x = 3\cos t$

$y = 3\sin t$

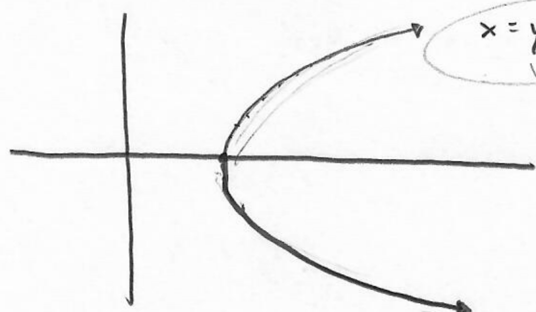
$0 \le t \le 2\pi$

$0 \le t \le \pi$

RESTRICT.

ex. FIND A PARAMETRIZATION FOR THE LOWER HALF OF THE PARABOLA

$x - 1 = y^2$.



$x = y^2 + 1$, $x - 1 = y^2$

$y = \pm\sqrt{x-1}$

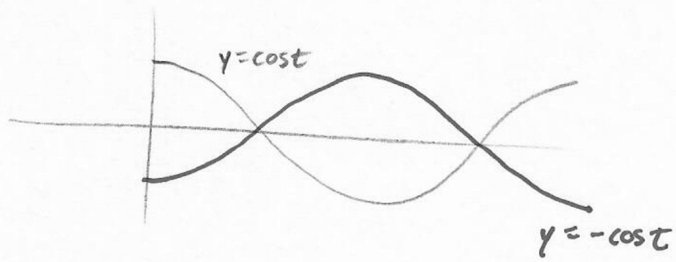
$x = y^2 + 1$

$y \le 0$ $y = -\sqrt{x-1}$

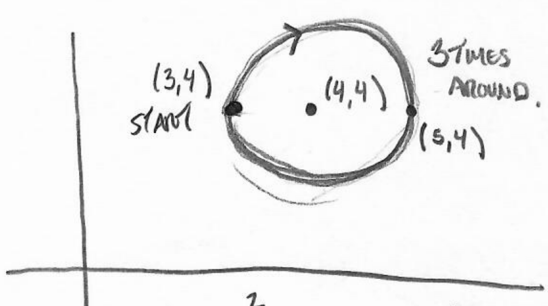
* EASY PARAMETRIZATION

$$\boxed{\begin{array}{l} * \; y = t \,, \quad t \le 0 \\ x = t^2 + 1 \end{array}}$$

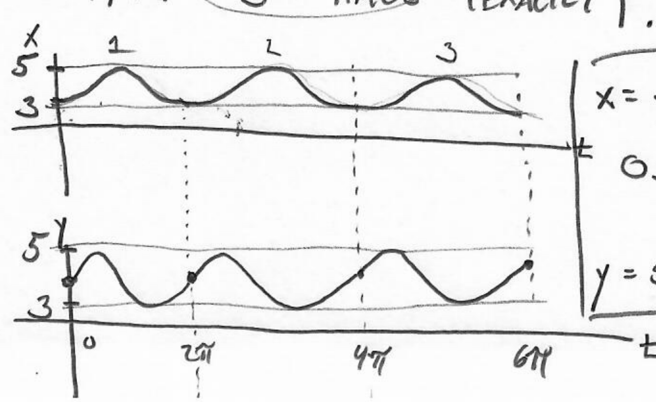
$$\boxed{\begin{array}{l} * \; x = t \,, \quad t \ge 1 \\ y = -\sqrt{t-1} \end{array}}$$



ex. FIND PARAMETRIC EQ'S FOR MOTION OF A PARTICLE THAT STARTS AT (3,4) AND MOVES CLOCKWISE AROUND A UNIT CIRCLE CENTERED AT (4,4) 3 TIMES (EXACTLY).



$(x-4)^2 + (y-4)^2 = 1$



$$\boxed{\begin{array}{l} x = -\cos t + 4 \\ 0 \le t \le 6\pi \\ y = \sin t + 4 \end{array}}$$

$0 \le t \le \pi$ ($\frac{1}{2}$x around)

$\cos t = x$

$\sin t = y$

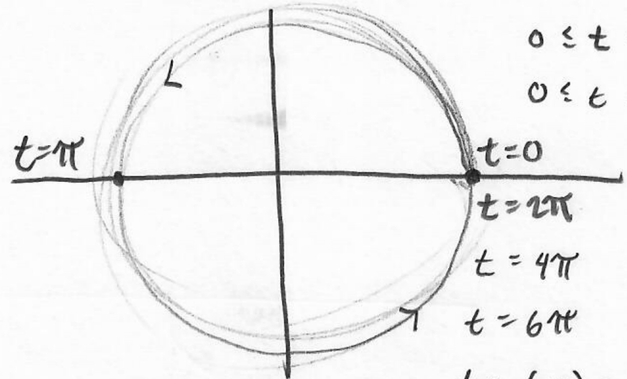
~~$a \le t \le b$~~

$-\infty < t < \infty$

$0 \le t \le 2\pi$ once around

$0 \le t \le 4\pi$ 2x around

$0 \le t \le 6\pi$ 3x around

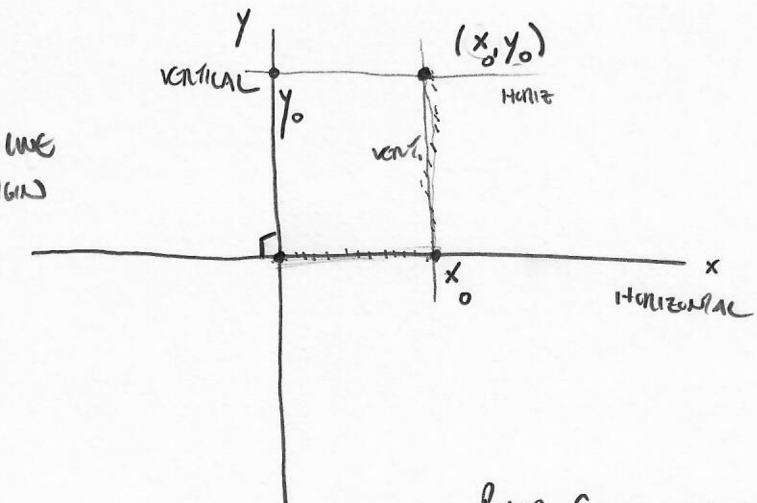


$t=(2\pi)n$, $n=1,2,\dots$

§11.3 Polar Coordinates

Coordinate systems: Cartesian coordinate system

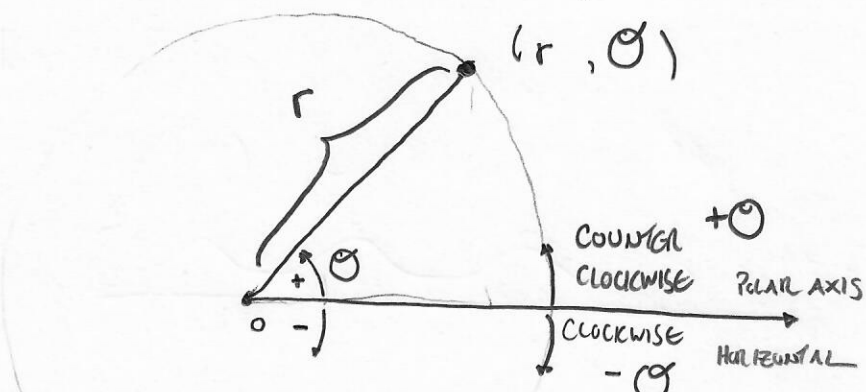
- Origin
- X-axis } Number line thru-origin
  Y-axis } ⊥



Polar Coordinates.

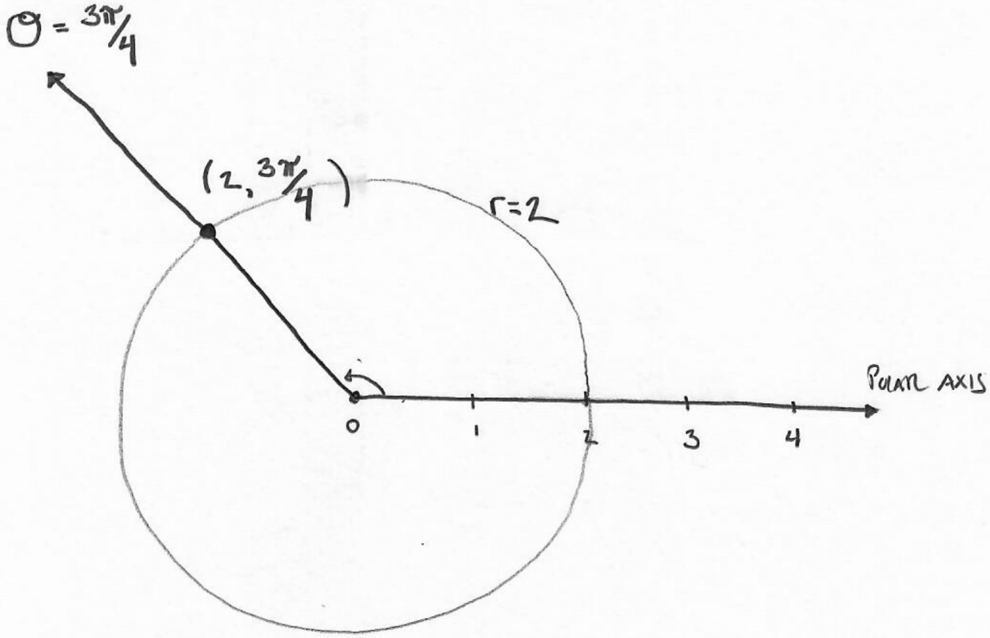
Polar coordinate system

- Origin
- Polar axis (ray beginning at origin)



θ measures rotation from polar axis

Plot: $\left(2, \frac{3\pi}{4}\right)$



NOTE: • ORIGIN DOES NOT HAVE A UNIQUE θ- COORDINATE

i.e. POLAR COORD. FOR ORIGIN $(0, \theta)$

θ ANY VALUE

• GIVEN ANY POINT ITS POLAR COORDINATES ARE NOT UNIQUE.

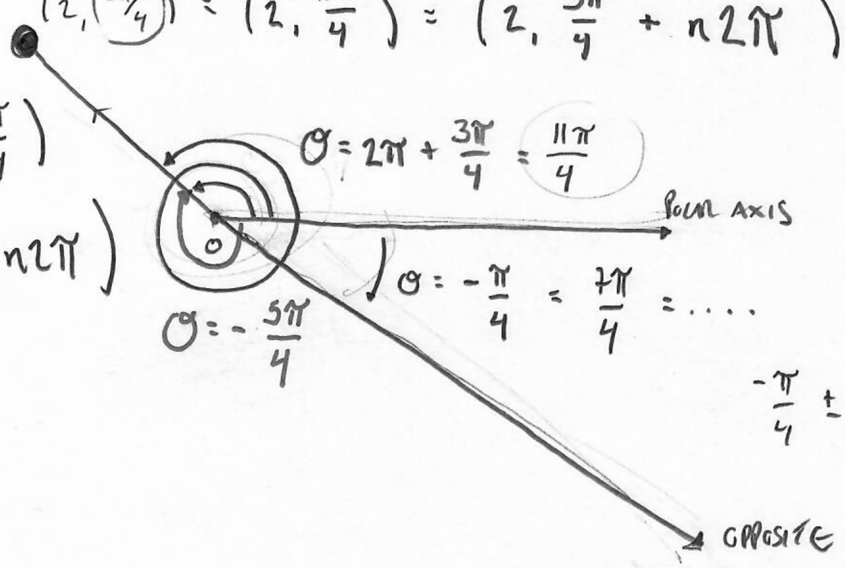
-2π

$= \left(2, -\frac{5\pi}{4}\right)$

$\left(2, 3\pi/4\right) = \left(2, \frac{11\pi}{4}\right) = \left(2, \frac{3\pi}{4} + n2\pi\right) \quad n = 0, \pm 1, \pm 2, \ldots$

$\left(-2, -\frac{\pi}{4}\right)$

$= \left(-2, -\frac{\pi}{4} \pm n2\pi\right)$



$\theta = 2\pi + \frac{3\pi}{4} = \frac{11\pi}{4}$

$\theta = -\frac{\pi}{4} = \frac{7\pi}{4} = \ldots$

$-\frac{\pi}{4} \pm n2\pi$

$\theta = -\frac{5\pi}{4}$