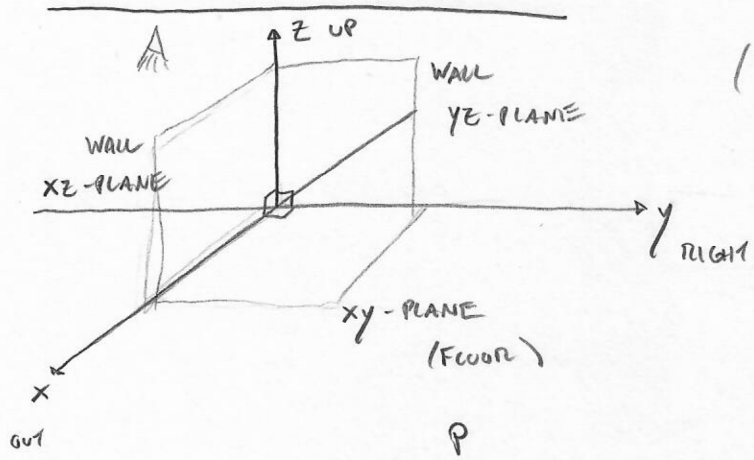
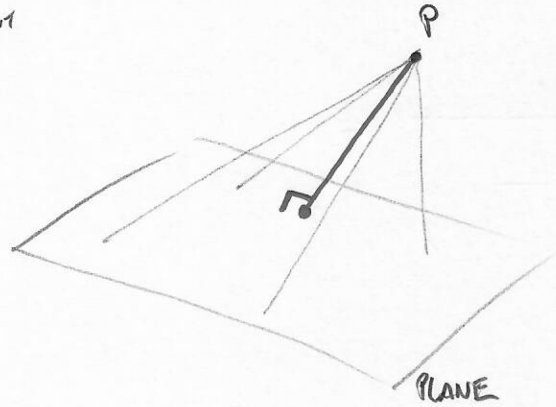
§12.1 3D COORD. SYSTEM

(RIGHT-HAND RULE)



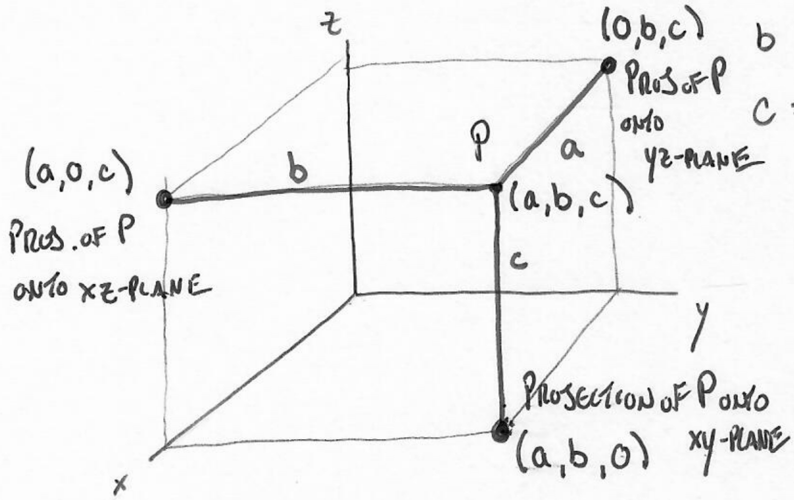


DISTANCE FROM POINT P TO A PLANE?

LENGTH OF LINE SEGMENT FROM P TO PLANE THAT HITS THE PLANE AT A RIGHT ANGLE.

GIVEN A POINT IN SPACE, LET

a = DIST TO THE YZ-PLANE (X-COORD)
b = DIST. TO THE XZ-PLANE (Y-COORD)
c = DIST TO THE XY-PLANE (Z-COORD)

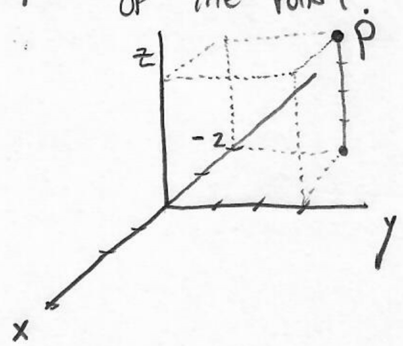


a, b, c ARE CALLED THE <u>COORDINATES</u> OF A POINT.

THE COORDINATES DETERMINE THE LOCATION OF THE POINT.

<u>ex.</u> $P(-2, 3, 4)$

$\uparrow$ x, $\uparrow$ y, $\uparrow$ z



GRAPHS: GIVEN AN EQUATION IN 3 VARIABLE x, y, z, ITS GRAPH IS THE SET OF ALL POINTS (x, y, z) THAT SATISFY THE EQUATION.

e.g. $x + y = z$ ← LIST SOME POINTS ON THIS GRAPH

(PLANE) $(1, 1, 2)$, $(0, 0, 0)$, ETC.

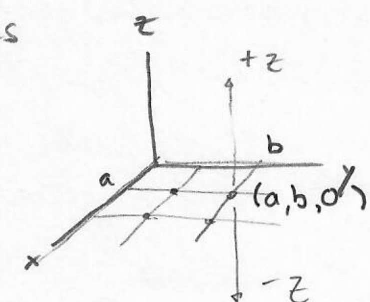
$x\ \ y\ \ z$

e.g. $x^2 + y^2 = z^2$ $(0,0,0)$, $(1,1,\sqrt{2})$,

(CONE) $(3,4,5)$, ETC.

- THE GRAPH OF AN EQUATION IN 2 VAR x, y IS A CURVE.
- THE GRAPH OF AN EQUATION IN 3 VAR x, y, z IS A SURFACE.

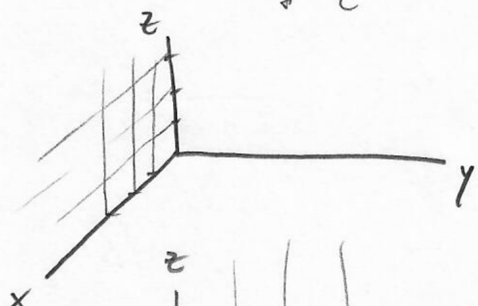
NOTE: THE EQUATIONS OF THE COORDINATE PLANES



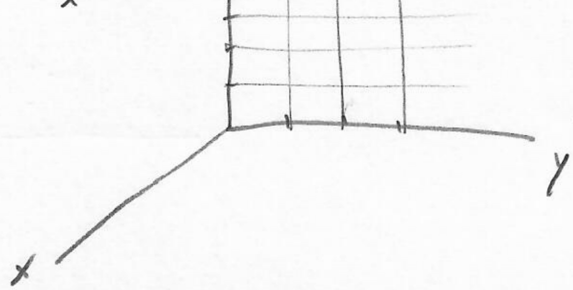
xy-PLANE: $z = 0$

↑ "DISTANCE" TO xy-PLANE

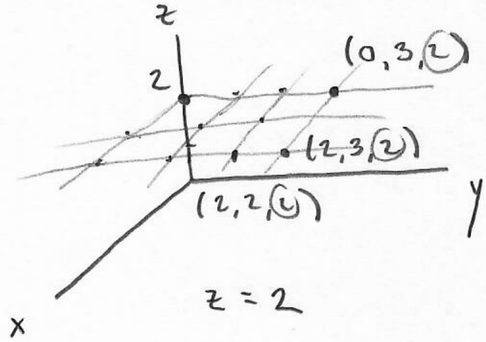
xz-PLANE: $y = 0$



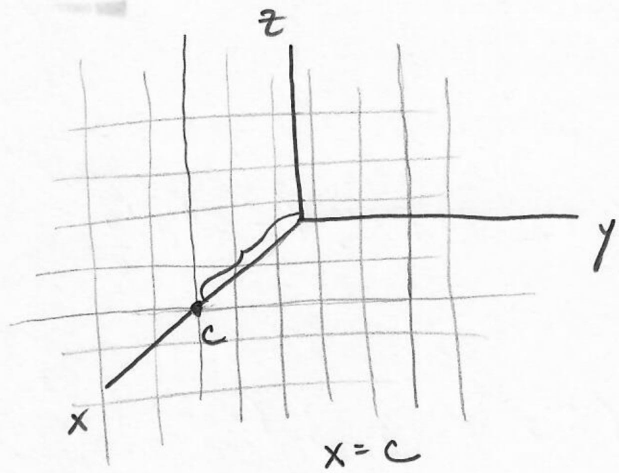
yz-PLANE $x = 0$



PLANES PARALLEL TO $\overbrace{\text{COORDINATE}}^{c = 0}$ PLANES: $x = c$, $y = c$, $z = c$



$z = 2$



$x = c$

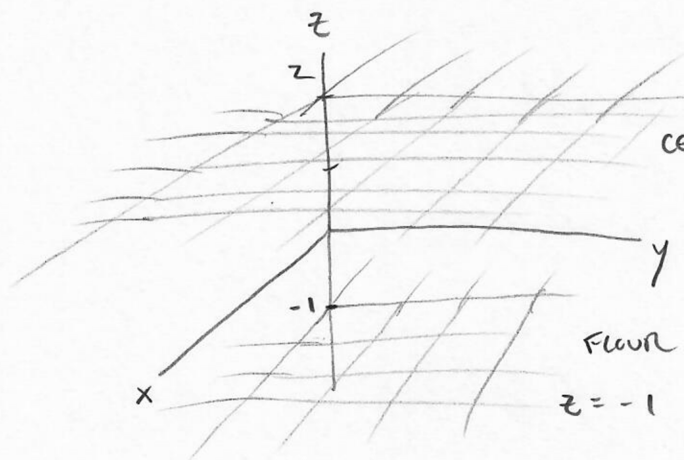
($\parallel$ to yz - PLANE

INEQUALITIES :

$-1 \leq z \leq 2$

$(1, 2, 3)$ ⊗

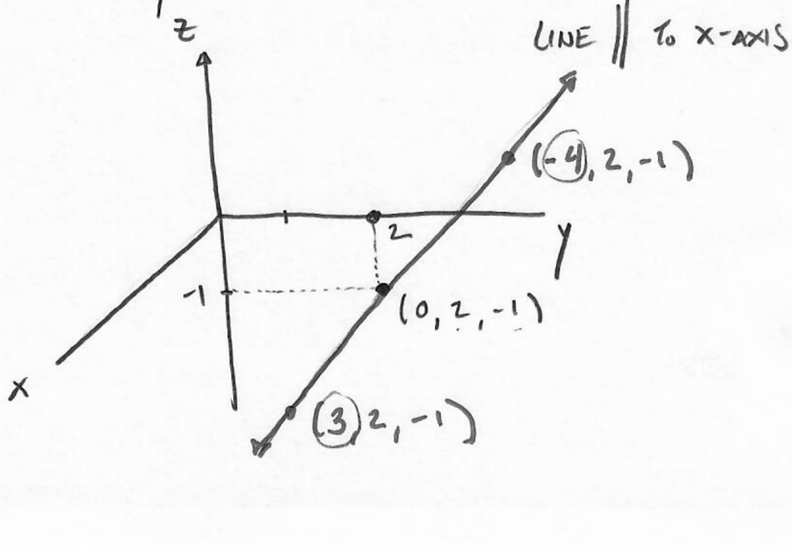
$(3, 2, 1)$ ✓



← SLAB BETWEEN PLANES $z = -1$, $z = 2$, INCLUDING THE PLANES.

e.g. SKETCH ALL POINTS SATISFYING $y = 2$, $z = -1$.

$(x, 2, -1)$

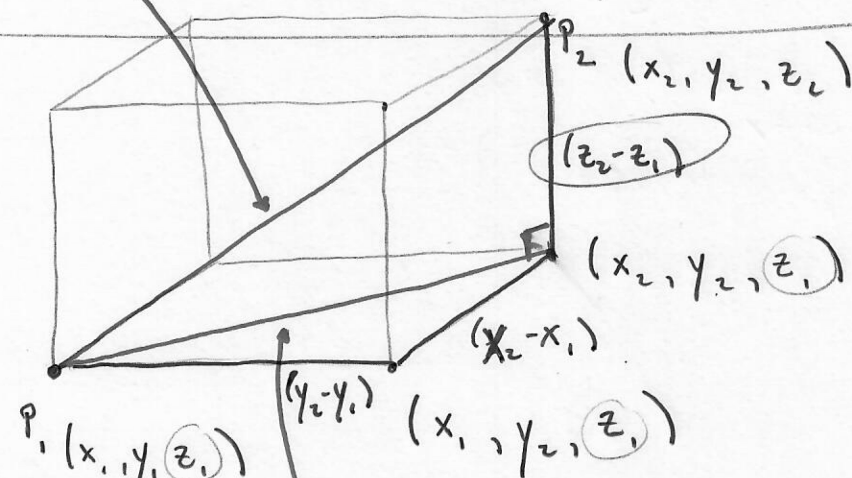


DISTANCE FORMULA IN 3 DIMENSIONS:

GIVEN $P_1(x_1, y_1, z_1)$ & $P_2(x_2, y_2, z_2)$,

THE DISTANCE BETWEEN P_1 & P_2 IS

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$



$$\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

e.g. FIND DISTANCE BETWEEN $(6, -5, 9)$ & $(-2, 4, -3)$

$x_1 \quad y_1 \quad z_1 \qquad x_2 \quad y_2 \quad z_2$

$$d = \sqrt{(-2-6)^2 + (4+5)^2 + (-3-9)^2}$$

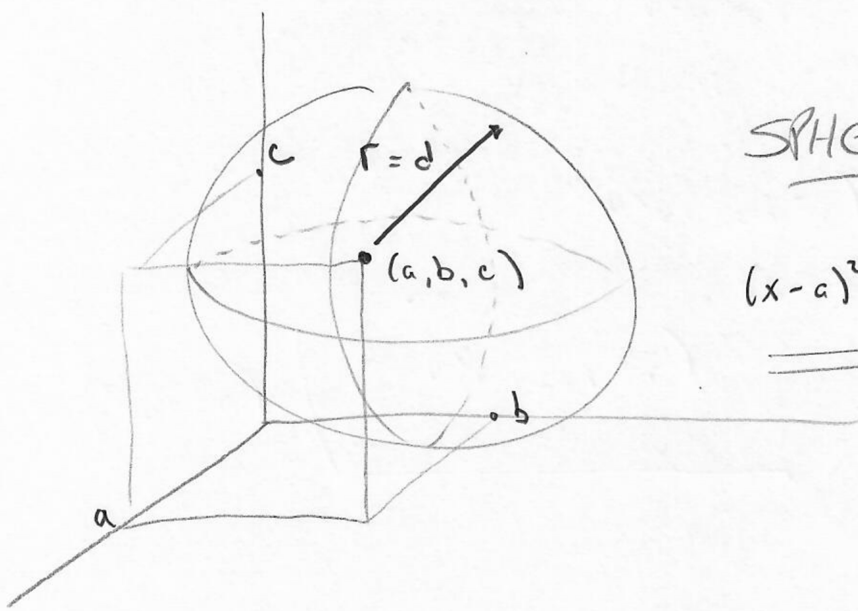
$$d = \sqrt{(-8)^2 + (9)^2 + (12)^2} = \underline{17}.$$

WHAT POINTS WOULD SATISFY THE EQUATION (a, b, c, d ALL CONSTANT)

$$d = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}, \quad d \geq 0$$

DISTANCE BETWEEN (x, y, z) & (a, b, c) IS d.

ALL POINTS DISTANCE d FROM THE POINT (a, b, c).



SPHERE

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$

ex. SHOW THAT $x^2 + y^2 + z^2 + 8x - 6y + 2z + 17 = 0$

REPRESENTS A SPHERE, FIND ITS CENTER AND RADIUS.

① ~ $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$

$$x^2 + 8x + 16 + y^2 - 6y + 9 + z^2 + 2z + 1 = -17 + 16 + 9 + 1$$

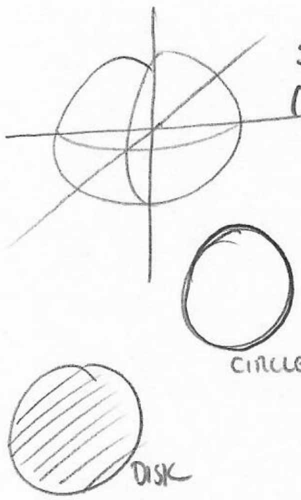
$$(x+4)^2 + (y-3)^2 + (z+1)^2 = 9 = 3^2$$

SPHERE W/ CENTER $(-4, 3, -1)$

RADIUS: 3

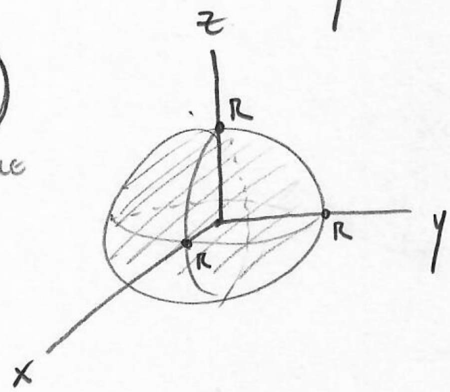
$(0,0,0)$

e.g. DESCRIBE THE SOLID BALL OF RADIUS R CENTERED AT ORIGIN.



SPHERE: (SHELL) $(x-0)^2 + (y-0)^2 + (z-0)^2 = R^2$

$$x^2 + y^2 + z^2 = R^2$$



EXTERIOR

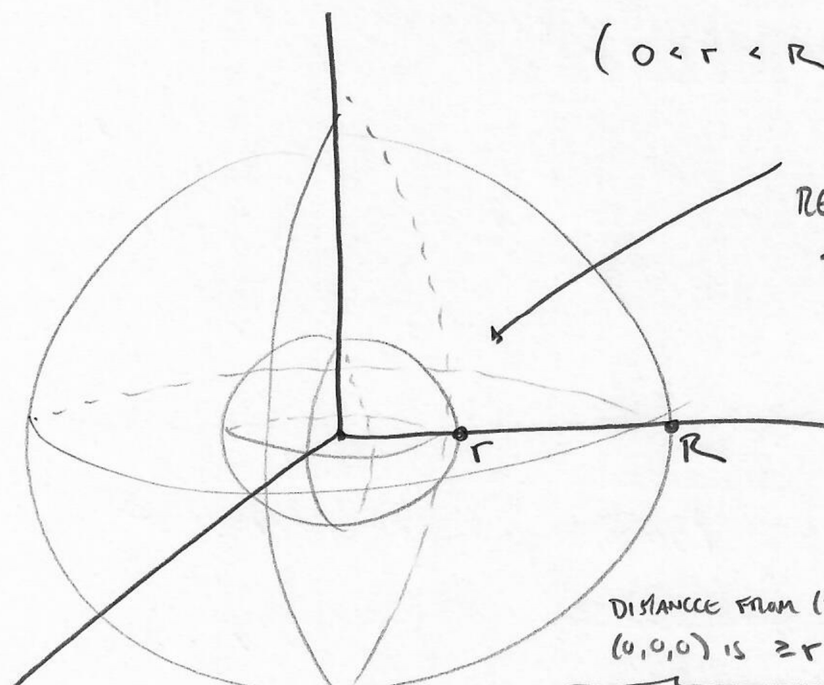
$>$

$$x^2 + y^2 + z^2 \leq R^2$$

SOLID BALL

WHAT ABOUT $\overbrace{r^2 \leq x^2 + y^2 + z^2}^{\text{EXT.}} \leq R^2$, where $\underbrace{x^2 + y^2 + z^2 \leq R^2}_{\text{INT.}}$

$(0 < r < R)$



REGION BETWEEN THE 2 SPHERES

DISTANCE FROM (x,y,z) TO $(0,0,0)$ IS $\geq r$

$$\overbrace{r \leq \sqrt{x^2+y^2+z^2}}^{\text{distance from } (x,y,z) \text{ to } (0,0,0) \text{ is } \geq r} \leq R, \qquad \underbrace{\sqrt{x^2+y^2+z^2} \leq R}_{\text{distance from } (x,y,z) \text{ to } (0,0,0) \text{ is } \leq R}$$

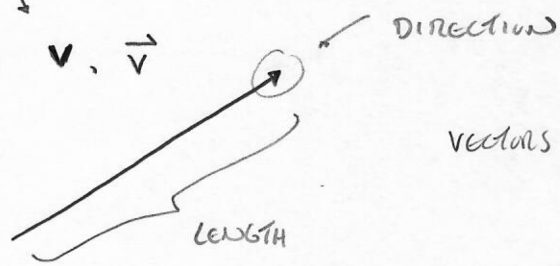
DISTANCE FROM (x,y,z) TO $(0,0,0)$ IS $\leq R$

§12.2 VECTORS

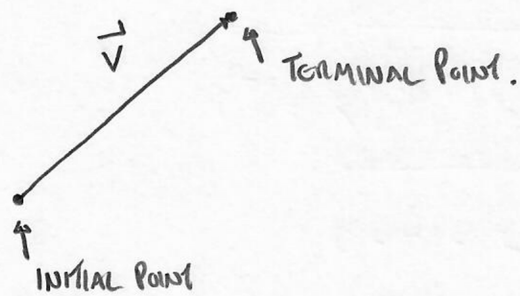
A VECTOR IS A GEOMETRIC OBJECT WITH

1. MAGNITUDE (SIZE) → LENGTH
2. DIRECTION.

BOLD
↓
DENOTED **v**, $\vec{v}$

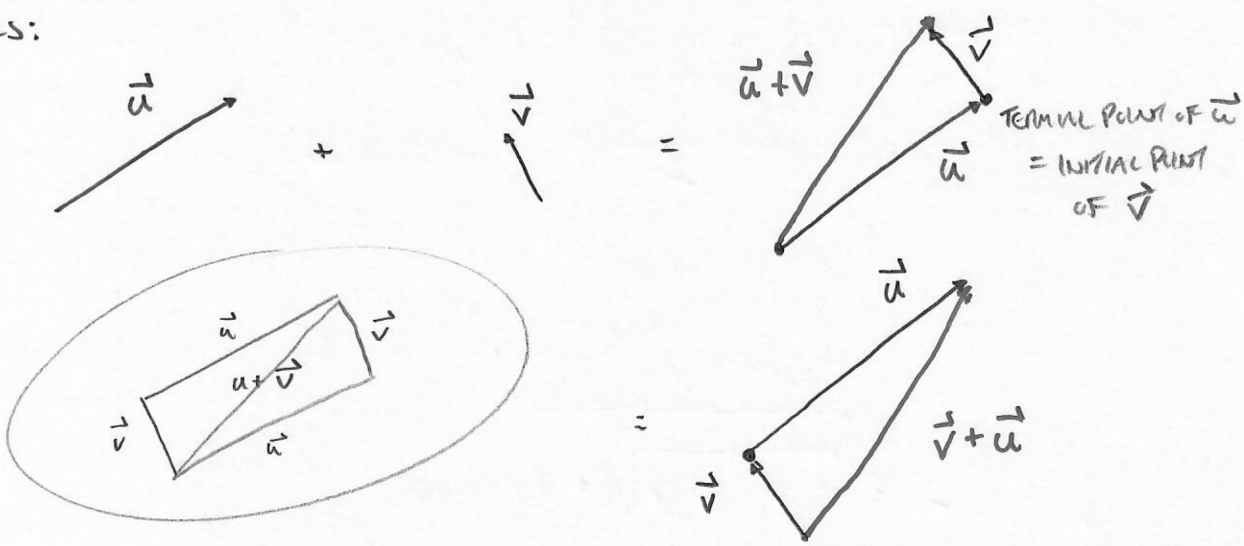


VECTORS CORRESPOND TO "ARROWS"



CONCATINATE

ADDING VECTORS:



Note: $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

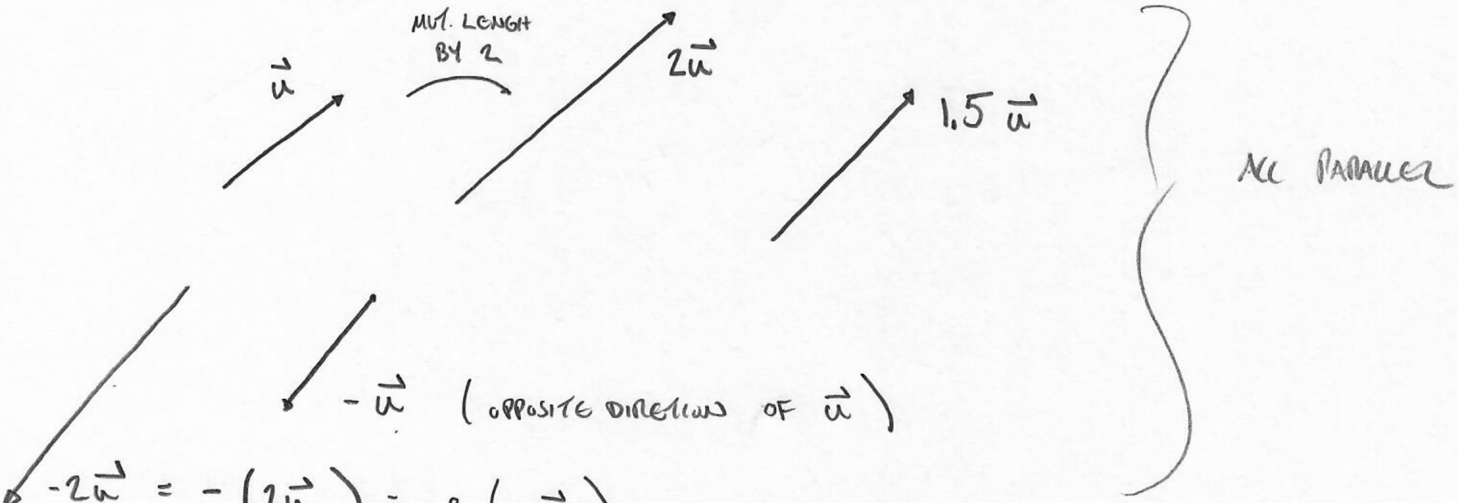
WE UNDERSTAND $\vec{u}$, $2\vec{u} = \vec{u} + \vec{u}$, $3\vec{u} = \vec{u} + \vec{u} + \vec{u}$, ETC...

WHAT ABOUT $c\vec{u}$ WHERE $c \in \mathbb{R}$.

SCALAR MULTIPLICATION

↳ SCALE THE VECTOR $\vec{u}$ BY c.

(c IS <u>NOT</u> A VECTOR ⟹ CALLED A SCALAR)



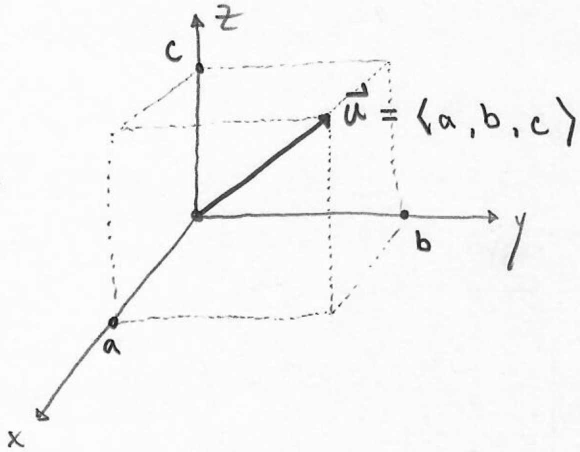
$-2\vec{u} = -(2\vec{u}) = 2(-\vec{u})$

NOTE: FOR ANY VECTOR $\vec{u}$, $0\vec{u}$ IS THE ZERO VECTOR (DEGENERATE)

- LENGTH 0, DIRECTION UNDEFINED.

BY INTRODUCING A <u>COORDINATE SYSTEM</u>, VECTORS ARE ALSO ALGEBRAIC OBJECTS.

3 DIM.



·) PLACE THE ORIGIN AT INITIAL POINT OF VECTOR

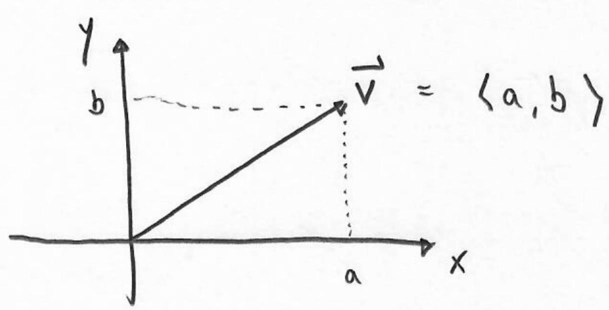
·) NOW THE VECTOR CAN BE DESCRIBED BY THE <u>COORDINATES</u> OF <u>THE TERMINAL POINT</u>, CALLED THE COMPONENTS OF THE VECTOR.

ANGLED BRACKETS FOR COMPONENTS.

$\vec{u} = \langle a, b, c \rangle$ REPRESENTS THE VECTOR $\vec{u}$ IN COMPONENT - FORM

x, y, z COMPONENTS OF $\vec{u}$

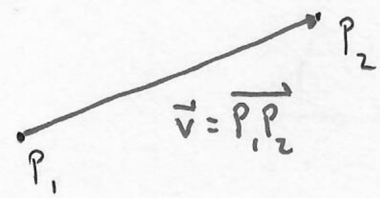
2 DIM.



GIVEN $P_1(x_1, y_1, z_1)$ & $P_2(x_2, y_2, z_2)$

<u>THE VECTOR $\vec{v}$ REPRESENTED BY $\overrightarrow{P_1P_2}$</u> IS

$$\vec{v} = \langle x_2 - x_1 , y_2 - y_1 , z_2 - z_1 \rangle$$



COMPONENT FORM: COORD. OF TERM. POINT. WHEN INITIAL POINT IS ORIGIN.

GIVEN THE VECTOR $\vec{v} = \langle x , y \rangle$ OR $\vec{v} = \langle x, y, z \rangle$

THE MAGNITUDE OF $\vec{v}$ (LENGTH OF $\vec{v}$) IS DENOTED

$|\vec{v}| = \|\vec{v}\|$ = DISTANCE FROM TERMINAL POINT OF $\vec{v}$ TO THE INITIAL <u>POINT OF $\vec{v}$</u> (ORIGIN)

$$= \begin{cases} \sqrt{x^2 + y^2} & \text{IN 2 DIM} \\ \sqrt{x^2 + y^2 + z^2} & \text{IN 3 DIM.} \end{cases}$$

ZERO VECTOR

NOTE: $|\vec{v}| \geq 0$ AND $|\vec{v}| = 0 \iff \vec{v} = \vec{0}$

$$= \begin{cases} \langle 0, 0 \rangle & 2D \\ \langle 0, 0, 0 \rangle & 3D \end{cases}$$

§ 12.1 , 12.2 (START)

§ 11.1 , 11.3