$$\boxed{\cosh^2 x - \sinh^2 x = 1}$$

$$\left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$$

$$\left(\frac{\cancel{e^{2x}} + 2 + \cancel{e^{-2x}}}{4}\right) - \left(\frac{\cancel{e^{2x}} - 2 + \cancel{e^{-2x}}}{4}\right)$$

$$= \frac{4}{4} = \underline{1}$$

$$\frac{d}{dx} \cosh x = \sinh x$$

$$\frac{d}{dx} \sinh x = \cosh x$$

$\int \sin^6 x \cos^3 x$

$\cos^2 x$

$\int \sin^6 x \, (1-\sin^2 x) \cos x \, dx$

$\int u^6 (1-u^2) \, du$ $\quad u = \sin x$

$du = \cos x \, dx$

$$\boxed{\sin^2 x = \frac{1}{2}(1-\cos 2x)}$$

$$\cos^2 x = \frac{1}{2}(1+\cos 2x)$$

$\int \sin^2 x \, dx$ $\qquad \int \cos^2 x \, dx$

$\int \sin^4 x \, dx$

$$= \int (\sin^2 x)^2 \, dx = \int \left(\tfrac{1}{2}(1 - \cos 2x) \right)^2 dx$$

$$= \frac{1}{4} \int 1 - 2\cos 2x + \underline{\cos^2 (2x)} \, dx$$

$$= \frac{1}{4} \int 1 - 2\cos 2x + \overbrace{\tfrac{1}{2}(1 + \cos (4x))} \, dx$$

$$= \frac{1}{4} \int 1 - 2\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x \, dx$$

$$= \frac{1}{4} \left(x - \sin 2x + \frac{1}{2}x + \frac{1}{8}\sin 4x \right) + C$$

$$\sqrt{16 - 5x^2} \qquad u = \sqrt{5}\,x$$
$$u^2 \qquad du = \sqrt{5}\,dx$$

$$\boxed{\sqrt{a^2 - u^2} \rightsquigarrow \text{LET } u = a\sin\theta} \qquad du = a\cos\theta\, d\theta$$

$$\sqrt{a^2 - a^2\sin^2\theta} = a\cos\theta \checkmark$$

$$\boxed{\sqrt{u^2 - a^2} \rightsquigarrow \text{LET } u = a\sec\theta} \qquad du = a\sec\theta\tan\theta\, d\theta$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

$$\boxed{\tan^2\theta = \sec^2\theta - 1}$$

$$\sqrt{a^2\sec^2\theta - a^2} = a\tan\theta \checkmark$$

$$\boxed{\sqrt{u^2 + a^2} \longrightarrow \text{LET } u = a\tan\theta} \qquad du = a\sec^2\theta\, d\theta$$

$$\sqrt{a^2\tan^2\theta + a^2} = a\sec\theta \checkmark$$

---

e.g.

$$\frac{x^5 - 3x + 1}{(x-1)^2(x^2+3x+4)^2} \quad \text{PROPER} \checkmark$$

IRREDUCIBLE

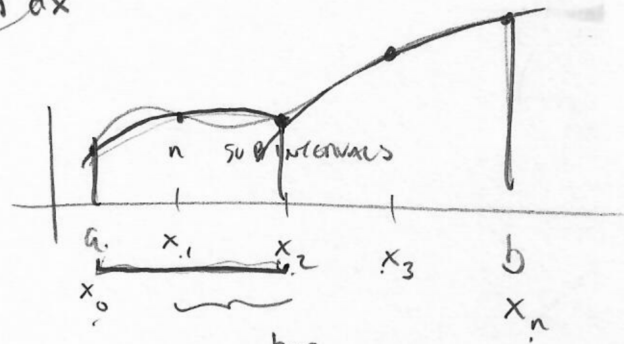
$b^2 - 4ac < 0$

$$= \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+3x+4} + \frac{Ex+F}{(x^2+3x+4)^2}$$

~~NEVER TOUCH IT~~!

$$\int_a^b f(x)\,dx$$

NO CALCULUS!



$$\Delta x = \frac{b-a}{n}$$

TRAP. RULE $$T_n = \frac{\Delta x}{2}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n) \right)$$

SIMPSON'S RULE $$S_n = \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-1}) + f(x_n) \right)$$

(n MUST BE EVEN)

$$\int_a^\infty f(x)\,dx = \underbrace{\lim_{t\to\infty} \underbrace{\int_a^t f(x)\,dx}_{\text{FIRST (ANSWER IN } t\text{)}}}_{\text{LAST}}$$

$$\int_{-\infty}^b \dots \qquad \int_{-\infty}^{\infty}$$

$$\underbrace{\int_{-\infty}^a \dots + \int_a^\infty \dots}$$

IF LIMIT DOESN'T EXIST

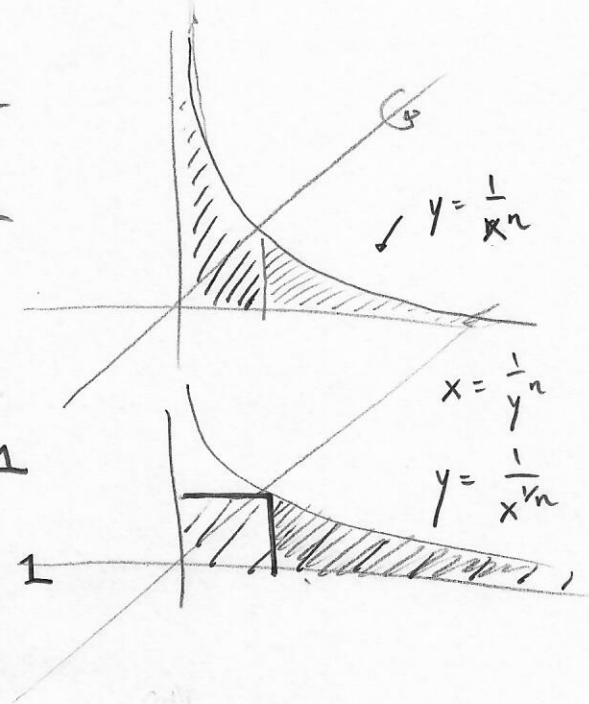
WE SAY IMP INT. DIVERGES.

IF IT DOES EXIST, THEN CONVERGES.

P - TEST

$$\int_a^\infty \frac{1}{x^n}\,dx \quad \begin{cases} \text{CONVERGES IF } n > 1 \\ \text{DIVERGES IF } n \le 1 \end{cases}$$

$a > 0$



ALT P - TEST

$$\int_0^a \frac{1}{x^n}\,dx = \begin{cases} \text{CONVERGES WHEN } n < 1 \\ \text{DIVERGES WHEN } n \ge 1 \end{cases}$$

$a > 0$

DIR. COMP THM $\quad 0 \le f(x) \le g(x)$

IF $\int_a^\infty g(x)\,dx$ CONV. THEN $\int_a^\infty f(x)\,dx$ CONV.

IF $\int_a^\infty f(x)$ DIVERGES THEN $\int_a^\infty g(x)$ DIVERGES.

LIMIT COMP THM $\quad$ IF $\lim_{x \to \infty} \frac{f(x)}{g(x)} = L$, $\quad 0 < L < \infty$

(ASSUME f, g BOTH POSITIVE)

THEN $\int_a^\infty f(x)\,dx$ AND $\int_a^\infty g(x)\,dx$

EITHER BOTH CONV. OR BOTH DIV.

Divergence Test

(n^{th} term test for divergence)

Limit of a sequence

Given $\sum_{n=1}^{\infty} a_n$, if $\lim_{n\to\infty} a_n \neq 0$

Then $\underbrace{\sum_{n=1}^{\infty} a_n}_{\text{Series}}$ diverges.

Limit of seq. of partial sums.

N^{th} partial sum

$$S_N = \sum_{n=1}^{N} a_n \rightarrow \sum_{n=1}^{\infty} a_n = \lim_{N\to\infty} S_N$$

Geometric Series: $\sum_{n=1}^{\infty} ar^{n-1} = \sum_{n=0}^{\infty} ar^n$

a = first term, r = common ratio

If $|r| < 1$ then Geom. Series conv. to $\dfrac{a}{1-r}$

Recursively defined sequences:

Prev. prev. term

Prev. term

$a_1 = 2$. $a_2 = 3$. For $n \geq 3$ $a_n = a_{n-2} - a_{n-1}$

2, 3, -1, 4, -5, 9, -14, 23, ...

SIN X

$$\int_0^1 f(x)\,dx$$

e.g 3

REPLACE f WITH n^{TH} ORDER TAYLOR POLYNOMIAL.

ALT SERIES

$$\int_0^1 x - \frac{x^3}{3!}\,dx = \frac{1}{2}x^2 - \frac{1}{4\cdot 3!}x^4$$

$$+\frac{x^5}{5!}$$

ESTIMATE THE ERROR

$$+\frac{x^6}{6\cdot 5!}$$

$$\frac{1}{2}x^2 - \frac{1}{4!}x^4 \Big|_0^1$$

$$\text{ERROR} \le \frac{1}{6\cdot 5!}$$

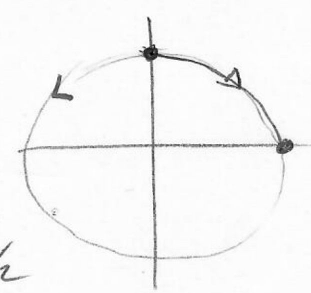
$$\frac{1}{2}x^2 - \frac{1}{4!}x^4 + \frac{1}{6!}x^6 \Big|_0^1$$

$x = \sin t$

$y = \cos t$

$0 \le t \le 2\pi$

$\pi/2$

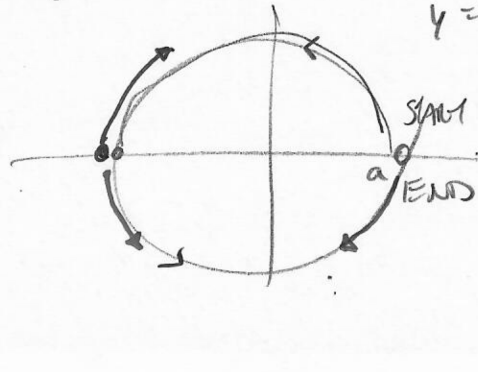


$$\begin{aligned} x &= f(t) \\ y &= g(t) \end{aligned}$$

POSITION OF PARTICLE AT TIME t

$x = a\cos t$

$y = a\sin t$



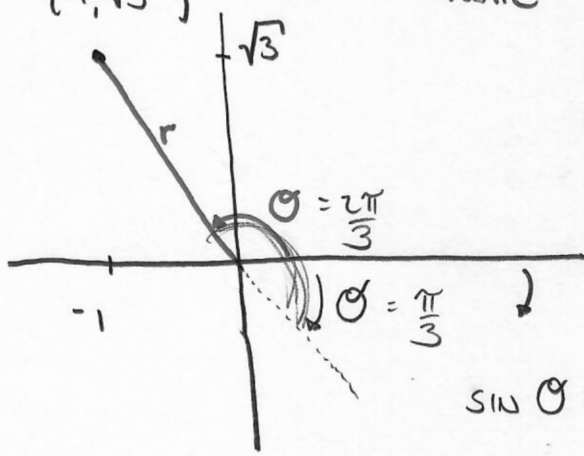
$0 \le t \le 2\pi$ 1x

$0 \le t \le 4\pi$ 2x

$0 \le t \le \pi$ $\frac{1}{2}$x

$$\left.\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}\right\}$$ POLAR $\rightarrow$ CART.

$(-1, \sqrt{3}) \longrightarrow$ POLAR (ASSUME $\overbrace{r \geq 0, \; 0 \leq \theta \leq 2\pi}^{\text{UNIQUE}}$)



$r^2 = x^2 + y^2$

$r = \sqrt{1+3} = 2$

$\sin\theta = \frac{y}{r} = \frac{\sqrt{3}}{2} \rightarrow \theta = \frac{\pi}{3} + n2\pi, \; \frac{2\pi}{3} + n2\pi$

$\cos\theta = \frac{x}{r} = \frac{-1}{2}$

$\hookrightarrow \theta = \pm\frac{2\pi}{3} + n2\pi$

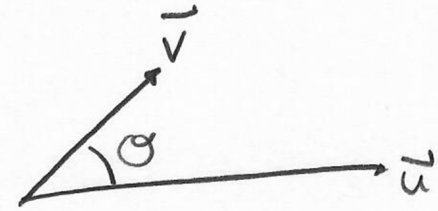
$(-1, \sqrt{3})_{\text{CART.}} = \left(2, \frac{2\pi}{3}\right)_{\text{POLAR}}$

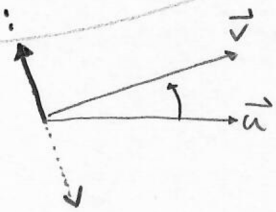
$= \left(2, \frac{2\pi}{3} + n2\pi\right)$

$= \left(-2, -\frac{\pi}{3}\right)$

$= \left(-2, -\frac{\pi}{3} + n2\pi\right)$

$\vec{u} = \langle x_1, y_1, z_1 \rangle$, $\vec{v} = \langle x_2, y_2, z_2 \rangle$



MULT	GEOM. DEF	ANALYTIC DEF
$\vec{u} \cdot \vec{v}$ DOT PROD, SCALAR PROD.	$\vec{u} \cdot \vec{v} = \lvert\vec{u}\rvert \lvert\vec{v}\rvert \cos\theta$	$x_1 x_2 + y_1 y_2 + z_1 z_2$ (SCALAR)
$\vec{u} \times \vec{v}$ CROSS PROD. VECTOR PROD.	LENGTH: $\lvert\vec{u} \times \vec{v}\rvert = \lvert\vec{u}\rvert \lvert\vec{v}\rvert \sin\theta$ DIRECTION: $\perp$ TO BOTH $\vec{u}, \vec{v}$ RHR: 	$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$ $= \langle y_1 z_2 - y_2 z_1 , z_1 x_2 - z_2 x_1 , x_1 y_2 - x_2 y_1 \rangle$

$\vec{u}, \vec{v}$ VECTORS. π_1 & π_2 ARE PLANES WITH NORMAL VECTORS $\vec{n}_1$ & $\vec{n}_2$

$\vec{u} \perp \vec{v} \iff \vec{u} \cdot \vec{v} = 0$

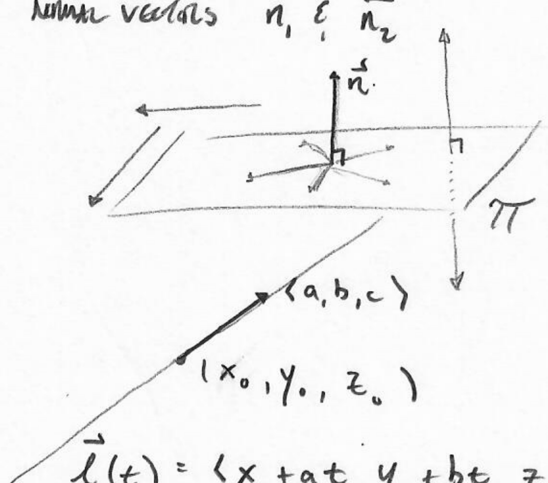
$\vec{u} \parallel \vec{v} \iff \vec{u} = c\vec{v}$

$\vec{u} \perp \pi_1 \iff \vec{u} \parallel \vec{n}_1$ $\quad (\iff \vec{u} = c\vec{n}_1$

$\vec{u} \parallel \pi_1 \iff \vec{u} \perp \vec{n}_1$ $\quad \iff \vec{u} \cdot \vec{n}_1 = 0)$

$\pi_1 \parallel \pi_2 \iff \vec{n}_1 \parallel \vec{n}_2$

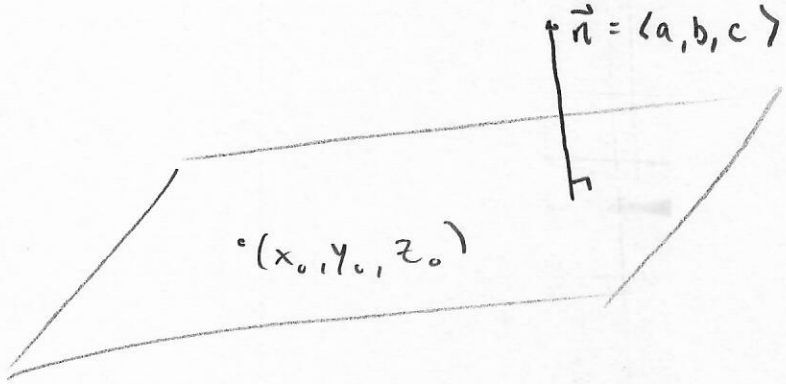
$\pi_1 \perp \pi_2 \iff \vec{n}_1 \perp \vec{n}_2$



$\vec{l}(t) = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle$

$x = x_0 + at$

$y = y_0 + bt$



EQ OF PLANE.

$$ax + by + cz = \underbrace{ax_0 + by_0 + cz_0}_{d}$$