Name:____ **Due 5/25/2017**

Math 392 Linear Algebra and Vector Analysis for Engineers **Quiz 5**

1 Part 1: Answer question 1

1. (8 points) (a) Calculate the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 2 & 5 \\ 4 & 1 \end{bmatrix}.$$

(b) Use part (a) to solve the system of ordinary differential equations

$$\begin{array}{rcrcr} y_1' & = & 2y_1 & + & 5y_2 \\ y_2' & = & 4y_1 & + & y_2 \end{array} \quad \text{with initial conditions} \quad \begin{array}{rcl} y_1(0) & = & 1 \\ y_2(0) & = & 7 \end{array}.$$

2 Part 2: Answer 3 out of 4 of the following questions.

2. (8 points) A particle starts at the point $(-2, 0)$, moves along the x-axis to $(2, 0)$, and then along the semicircle $y = \sqrt{4 - x^2}$ back to the starting point. Find the work done on this particle by the force field

$$\vec{F}(x, y) = \left\langle x, x^3 + 3xy^2 \right\rangle .$$

3. (8 points) Let

$$\vec{F} = x^2\vec{\imath} + (x+z)\vec{\jmath} + yz\vec{k},$$

let S bet the surface of the paraboloid $z = 4 - x^2 - y^2$ that lies above the xy-plane, and let $\vec{n}$ be the upward pointing unit normal vector to S. Find the value of the surface integral

$$\iint_S \operatorname{curl} \vec{F} \cdot \vec{n}\, dS$$

(a) directly, and

(b) using Stoke's theorem.

4. (8 points) Let

$$\vec{F} = x\vec{i} + y\vec{j} + z\vec{k},$$

let S be the surface of the region between the graphs of $z = \sqrt{2 - x^2 - y^2}$ and $z = \sqrt{x^2 + y^2}$, and let $\vec{n}$ be the outward pointing unit normal vector to S (Note that $\vec{n}$ points upward from the top surface and downward from the bottom surface). Find the value of the surface integral

$$\iint_S \vec{F} \cdot \vec{n}\, dS$$

(a) directly, and

(b) using the divergence theorem.

5. (8 points) Use Green's theorem to find the area under one arch of the cycloid given by the parametric equations

$$C_1 : \begin{array}{rcl} x(t) & = & t - \sin t \\ y(t) & = & 1 - \cos t \end{array} \quad ; \quad 0 \leq t \leq 2\pi.$$

That is, find the area of the region enclosed by $C = C_1 + C_2$, where C_2 is the line segment from $(2\pi, 0)$ to $(0, 0)$. Note that C is oriented *clockwise*.