§1.1 Functions & Their Representations

Def: A **function** is a rule that assigns to each **input** exactly one **output**.

(In this class: real numbers $\mathbb{R}$)

e.g. $f(x) = \frac{1}{x}$, $f(w) = \frac{1}{w}$ (same function - reciprocal)

The **domain** of a function is the set of all allowable inputs.

The **range** of a function f is the set of all possible outputs $f(x)$ as x varies throughout the domain.

ex. Describe the domain & range of

$$f(x) = \sqrt{x-5}, \quad g(x) = 2 + x^2$$

ex. Describe the domain of $f(t) = \dfrac{\sqrt{4-t}}{t^2-4}$.

■ EXAMPLE 6 A Function Defined by a Formula

If $f(x) = 2x^2 - 5x + 1$, evaluate

(a) $f(-3)$ **(b)** $f(4) - f(2)$ **(c)** $\dfrac{f(1+h) - f(1)}{h}$ $(h \neq 0)$

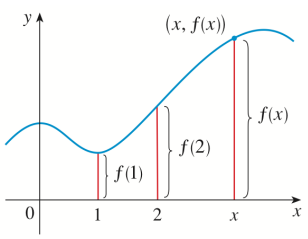
Piecewise-defined functions follow different rules for different inputs.

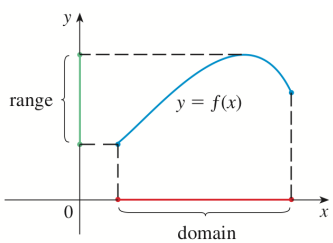
ex.
$$f(x) = \begin{cases} 2x-3 & \text{if } x \leq -1 \\ x^2 - 1 & \text{if } -1 < x \leq 2 \\ 8 & \text{if } x > 2 \end{cases}$$

What is $f(-2), f(0), f(2), f(4)$?

ex. $f(x) = |x|$ is a piecewise defined function.

Graphs The **graph** of f is the set of all points (x, y) such that $y = f(x)$.





ex. SKETCH GRAPH OF $f(x) = \begin{cases} 2x-3 & \text{IF } x \le -1 \\ x^2-1 & \text{IF } -1 < x \le 2 \\ 0 & \text{IF } x > 2 \end{cases}$

■ **The Vertical Line Test** A curve or scatter plot in the xy-plane is the graph of a function of x if and only if no vertical line intersects the graph more than once.

ex. SKETCH GRAPH OF TEMP OF WATER t SECONDS AFTER TURNING ON A HOT WATER FAUCET.

ODD & EVEN FUNCTIONS & THE SYMMETRY OF THEIR GRAPHS

Def: f IS EVEN IF $f(-x) = f(x)$ FOR ALL x IN DOMAIN

f IS ODD IF $f(-x) = -f(x)$ FOR ALL x IN DOMAIN

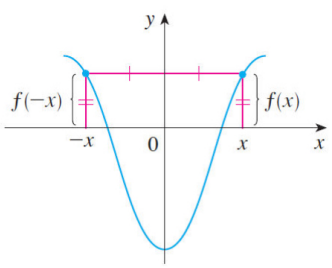


FIGURE 19 An even function

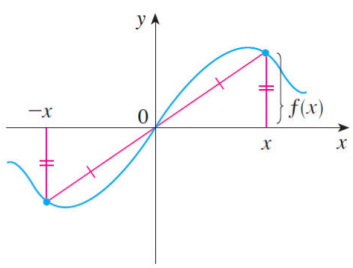


FIGURE 20 An odd function

57–62 ■ Determine whether f is even, odd, or neither. If you have a graphing calculator, use it to check your answer visually.

57. $f(x) = \dfrac{x}{x^2+1}$

58. $f(x) = \dfrac{x^2}{x^4+1}$

59. $f(x) = \dfrac{x}{x+1}$

60. $f(x) = x|x|$

61. $f(x) = 1 + 3x^2 - x^4$

62. $f(x) = 1 + 3x^3 - x^5$