§ 1.2 COMBINING & TRANSFORMING FUNCTIONS

■ EXAMPLE 2 Combining Revenue and Cost Functions

Suppose the annual revenue, in millions of dollars, of a company is $R(t) = 0.2t^2 + 3t + 5$, where t is measured in years and $t = 0$ corresponds to the year 2000. The annual cost, in millions of dollars, for the company is $C(t) = 4t + 9$.

(a) Find a formula for the function $P(t) = R(t) - C(t)$.

(b) Compute and interpret $P(7)$.

Def: GIVEN 2 FUNCTIONS f & g, THE COMPOSITION $f \circ g$ ("f OF g") IS THE FUNCTION $f \circ g(x) = f(g(x))$

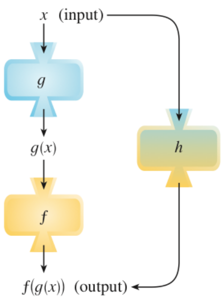


FIGURE 1
The h machine is composed of the g machine (first) and then the f machine.

ex. LET $f(x) = x^2 + 2x - 5$, $g(x) = 3x - 1$

FIND $f \circ g(x)$ & $g \circ f(x)$.

ex. LET $h(x) = \sqrt{5x - 1}$.

FIND f & g S.T. $h(x) = f(g(x))$.

(MORE THAN ONE WAY!)

15–20 ■ Find the functions $p(x) = f(g(x))$ and $q(x) = g(f(x))$.

15. $f(x) = x^2 - 1$, $g(x) = 2x + 1$

16. $f(x) = 1 - x^3$, $g(x) = 1/x$

17. $f(x) = x^3 + 2x$, $g(x) = 1 - \sqrt{x}$

18. $f(x) = 1 - 3x$, $g(x) = 5x^2 + 3x + 2$

19. $f(x) = x + \dfrac{1}{x}$, $g(x) = x + 2$

20. $f(x) = \sqrt{2x + 3}$, $g(x) = x^2 + 1$

DISCUSS

Car maintenance If $C(m)$ is the average annual cost for maintaining a Honda Civic that has been driven m thousand miles and $f(t)$ is the number of miles on Sean's Honda Civic t years after he purchased it, what does the function $g(t) = C(f(t))$ represent?

Carpooling As fuel prices increase, more drivers carpool. The function $f(p)$ gives the average percentage of commuters who carpool when the cost of gasoline is p dollars per gallon. If $g(t)$ is the average monthly price (per gallon) of gasoline, where t is the time in months beginning January 1, 2011, which composition gives a meaningful result, $f(g(t))$ or $g(f(p))$? Describe what the resulting function measures.

■ EXAMPLE 4 Interpreting a Composition of Functions

The altitude of a small airplane t hours after taking off is given by $A(t) = -2.8t^2 + 6.7t$ thousand feet, where $0 \leqslant t \leqslant 2$. The air temperature in the area at an altitude of x thousand feet is $f(x) = 68 - 3.5x$ degrees Fahrenheit.

(a) What does the composition $h(t) = f(A(t))$ measure?

(b) Compute $h(1)$ and interpret your result in this context.

(c) Find a formula for $h(t)$.

(d) Does $A(f(x))$ give a meaningful result in this context?

TRANSFORMATIONS OF FUNCTIONS

■ Vertical and Horizontal Shifts
Suppose c is a positive number.

translation of the graph of $y = f(x)$	equation
shift c units upward	$y = f(x) + c$
shift c units downward	$y = f(x) - c$
shift c units to the right	$y = f(x - c)$
shift c units to the left	$y = f(x + c)$

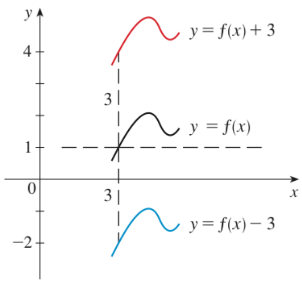


FIGURE 2
Vertical translations of the graph of f

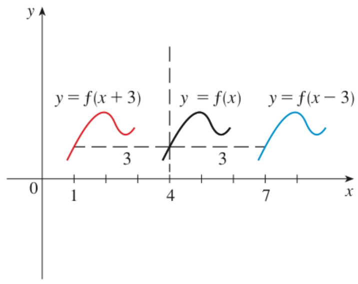
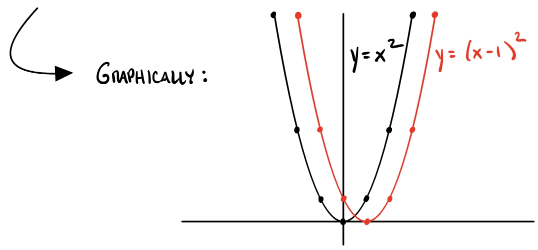


FIGURE 3
Horizontal translations of the graph of f

ex. COMPLETE THE TABLE OF VALUES:

	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$(x-1)^2$	16	9	4	1	0	1	4

GRAPHICALLY:



■ Vertical and Horizontal Stretching
Suppose $c > 1$.

transformation of the graph of $y = f(x)$	equation
stretch vertically by a factor of c	$y = cf(x)$
compress vertically by a factor of c	$y = \frac{1}{c}f(x)$
compress horizontally by a factor of c	$y = f(cx)$
stretch horizontally by a factor of c	$y = f\left(\frac{1}{c}x\right)$

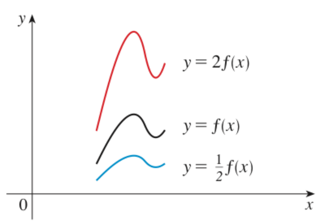


FIGURE 4
Stretching the graph of f vertically

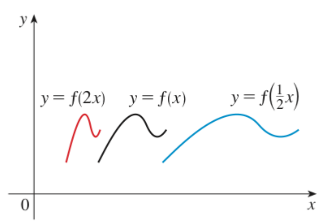
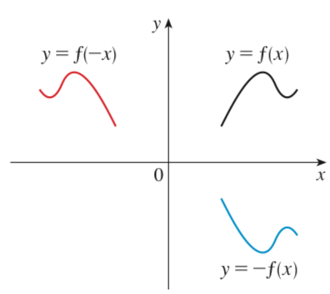


FIGURE 5
Stretching the graph of f horizontally

■ Vertical and Horizontal Reflections

reflection of the graph of $y = f(x)$	equation
reflect about the x-axis	$y = -f(x)$
reflect about the y-axis	$y = f(-x)$



38–42 ■ The graph of $y = \sqrt{x}$ is shown in Figure 8(a). Use transformations to graph each of the following functions.

38. $y = \sqrt{x} + 3$

39. $y = \sqrt{x + 3}$

40. $y = -\frac{1}{2}\sqrt{x}$

41. $y = -\sqrt{x - 1}$

42. $y = \sqrt{-x} + 2$

43–46 ■ The graph of $y = x^2$ is shown in Figure 9. Use transformations to graph each of the following functions.

43. $y = -x^2 + 2$

44. $y = (x - 1)^2 - 4$

45. $f(x) = \frac{1}{4}x^2 - 3$

46. $g(x) = -(x + 5)^2 + 3$