§1.5 EXPONENTIAL MODELS

SUPPOSE $\ell(x)$ IS A LINEAR FUNCTION
$f(x)$ IS AN EXPONENTIAL FUNCTION.

x	$\ell(x)$	$f(x)$
1	100	100
2	200	200
3	300	400
4	400	800
5	500	1600
6	600	3200

$+100$ ADDITION

$\times 2$ MULTIPLICATION

LINEAR $\ell(x) = mx + b$

$100 = m + b$
$200 = 2m + b$

$\Rightarrow \ell(x) = 100x$

DEF: $\boxed{\text{EXPONENTIAL } f(x) = ba^x}$

$100 = ba$
$200 = ba^2$

$\Rightarrow f(x) = 50 \cdot 2^x$

ex. SUPPOSE $f(8) = 250$ & $f(14) = 400$.

(a) FIND $f(20)$ IF f IS LINEAR / EXPONENTIAL. 550 / 640

(b) FIND $f(x)$ IF f IS LINEAR / EXPONENTIAL.

LINEAR: $f(x) = 25x + 50$

EXP: $f(x) = 250\,(1.6)^{(x-8)/6}$

ex. SKETCH THE GRAPH OF $y = 4^x$ BY PLOTTING POINTS.

If $0 < a < 1$, then a^x approaches 0 as x becomes large. If $a > 1$, then a^x approaches 0 as x decreases through negative values. In both cases the x-axis is a horizontal asymptote. These matters are discussed in Section 4.4.

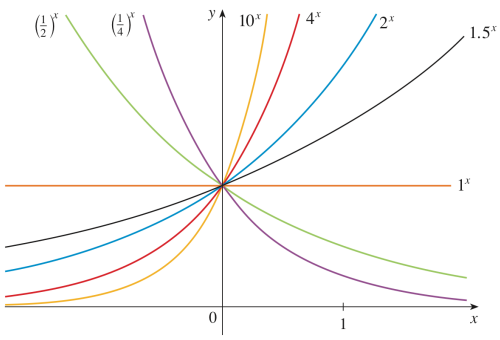

FIGURE 2

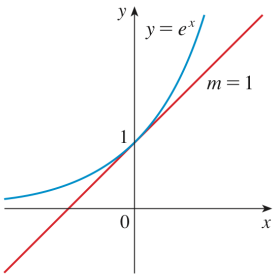

FIGURE 12
The natural exponential function crosses the y-axis with a slope of 1.

ex. SKETCH (a) $y = 5e^x$

(b) $y = e^{-x} + 2$

Laws of Exponents If a and b are positive numbers and x and y are any real numbers, then

1. $a^x \cdot a^y = a^{x+y}$ **2.** $\dfrac{a^x}{a^y} = a^{x-y}$ **3.** $(a^x)^y = a^{xy}$ **4.** $(ab)^x = a^x b^x$

EXAMPLE 3 Using Properties of Exponential Functions

For more review and practice using the Laws of Exponents, see Appendix A.

Show that each of the following is true.

(a) $8 \cdot (1.6)^{2x} = 8 \cdot (2.56)^x$

(b) $5 \cdot 4^{x/2} = 5 \cdot 2^x$

(c) $\dfrac{10}{5^{x/3}} = 10 \cdot (5^{-1/3})^x$ $\quad 2 \cdot 5 \cdot 5^{-\frac{1}{3}x} = 2 \cdot 5^{1-\frac{x}{3}}$

(d) $3^{4+2t} = 81 \cdot 9^t$

21–24 ■ Write each of the following as an expression using radicals.

21. $4^{2/3}$ **22.** $7^{5/2}$

23. $e^{1/4}$ **24.** $w^{3/4}$

25–30 ■ Show that each of the following statements is true.

25. $P \cdot 3^{3x} = P \cdot 27^x$ **26.** $8^{t/3} = 2^t$

27. $500 \cdot (1.025)^{4t} \approx 500 \cdot (1.1038)^t$

28. $\dfrac{1}{e^{x/2}} = \left(\dfrac{1}{\sqrt{e}}\right)^x$

29. $4^{x+3} = 64 \cdot 4^x$

30. $12e^{0.2t} \approx 12 \cdot (1.2214)^t$

ex. SUPPOSE A COLONY OF BACTERIA HAS INITIAL POPULATION 180 AND DOUBLES EVERY 5 HRS. FIND POPULATION $P(t)$, WHERE t IS HOURS AFTER INITIAL COUNT.

$$P(t) = 180 \cdot 2^{t/5}$$