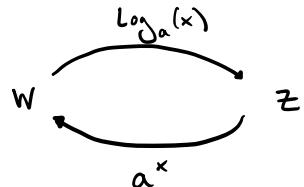


§1.6 Logarithmic Functions

Def: $\text{Log}_a(w) = z \Leftrightarrow a^z = w$

ex. $\text{Log}_2 16 = 4$ $\text{Log}_2 \frac{1}{4} = -2$
 $\text{Log}_9 3 = \frac{1}{2}$ $\text{Log}_{127} 1 = 0$
 $\text{Log}_4 8 = \frac{3}{2}$ $\text{Log}_{25} \frac{1}{5} = -\frac{1}{2}$

This says that the logarithm-base-a function is the **inverse** (opposite) of the exponential-base-a function

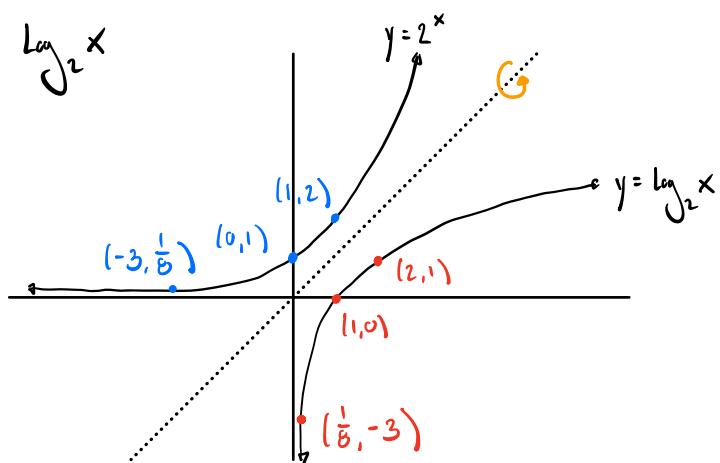


From DEF, IF $y = a^x$ THEN $x = \text{Log}_a y$

so IF (z, w) is on graph $y = a^x$ ($w = a^z$)

THEN (w, z) is on graph $y = \text{Log}_a x$ ($z = \text{Log}_a w$)

ex. Sketch $y = \text{Log}_2 x$



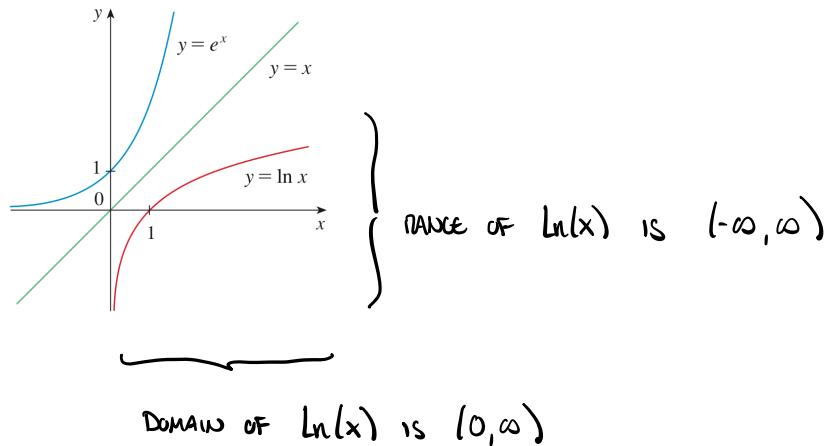
BASE 10 "common logarithm" $\log x = \log_{10} x$

BASE e "natural logarithm" $\ln x = \log_e x$

our focus

$$\ln(w) = z \Leftrightarrow e^z = w$$

$$\ln(1) = 0, \quad \ln(e) = 1$$



CANCELLATION Properties: $\ln(e^x) = x$ FOR ALL x

$$e^{\ln(x)} = x \quad \text{IF ALL } x > 0$$

■ Laws of Logarithms If x and y are positive numbers, then

1. $\ln(xy) = \ln x + \ln y$

2. $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

3. $\ln(x^r) = r \ln x$ (where r is any real number)

Proof: Let $x = e^w, y = e^z$ ($w = \ln x, z = \ln y$)

1: $\ln(xy) = \ln(e^w e^z) = \ln(e^{w+z}) = w + z = \ln x + \ln z$ ✓

2: $\ln\left(\frac{x}{y}\right) = \ln\left(\frac{e^w}{e^z}\right) = \ln(e^{w-z}) = w - z = \ln x - \ln z$ ✓

3: $\ln(x^r) = \ln(e^{wr}) = rw = r \ln x$ ✓

ex. Rewrite $\ln\left(\frac{x^2y^3}{\sqrt{z}}\right)$

33–34 ■ Solve each equation for x . Give both an exact solution and a decimal approximation, rounded to four decimal places.

33. (a) $2 \ln x = 1$

(b) $e^{-x} = 5$

34. (a) $e^{2x+3} - 7 = 0$

(b) $\ln(5 - 2x) = -3$

35–42 ■ Solve each equation. Give a decimal approximation, rounded to four decimal places.

35. $5^t = 20$

36. $1.13^x = 7.65$

37. $2^{x-5} = 3$

38. $10^{3-2x} = 42$

39. $8e^{3x} = 31$

40. $450e^{0.15t} = 1200$

41. $6 \cdot (2^{x/7}) = 11.4$

42. $100 \cdot (4^{-p/5}) = 8.8$

47. Bacteria population If a bacteria population starts with 100 bacteria and doubles every three hours, then the number of bacteria after t hours is

$$n = f(t) = 100 \cdot 2^{t/3}$$

(see Exercise 35 in Section 1.5). When will the population reach 50,000?

ex. $\ln(x) + 2 \ln(3x) = 81$