Exam 2

Answer all 10 questions for a total of 100 points. Write your solutions in the accompanying blue book, and put a box around your final answers. Your answers may be left as expressions involving square roots, logarithms, exponentials, etc. If you solve the problems out of order, please skip pages so that your solutions stay in order. Good luck!

1. Find the derivative.

- (a) (8 points) $f(x) = \sqrt{x} \ln(x^2 + 1)$
- (b) (8 points) $g(x) = \frac{e^{8x}}{(3x^2 + 5)^2}$ (c) (8 points) $h(x) = 120 \left(\frac{1}{3}\right)^{x/4}$
- 2. (8 points) A caterer estimates that it costs

$$C(q) = .02q^2 + 8q + 375$$

dollars to provide dinner to q guests at a charity event. How large of an event (how many guests) will minimize the average cost per guest?

- 3. (4 points) Suppose C(q) is the cost a company must pay to produce q units. If C(10, 500) = 8,200 and C'(10,500) = 0.5, approximately how much would it cost the company to produce 11,000 units?
- 4. Evaluate the following.
 - (a) (2 points) $\log_2\left(\frac{1}{8}\right)$ (b) (2 points) $\log_{64}(8)$
- 5. (10 points) Give an equation for the tangent line to the curve

$$x^2 + 4xy + y^2 = 13$$

at the point (2,1).

- 6. Suppose \$3,000 is invested into an account that earns an annual interest rate of 6%. What is the value of the investment at the end of 5 years ...
 - (a) (4 points) if the interest is compounded monthly?
 - (b) (4 points) if the interest is compounded continuously?
- 7. (8 points) Let C(t) be the concentration of a drug in the bloodstream. As the body eliminates the drug, C(t) decreases at a rate that is proportional to the amount of the drug that is present at the time. That is, C(t) obeys the law of natural growth/decay. If the body eliminates half the drug in 30 hours, how long does it take to eliminate 90% of the drug?
- 8. (10 points) A balloon is rising at a constant speed of 6 ft/s. A girl is cycling along a straight road at a speed of 18 ft/s. When she passes under the balloon, it is 54 ft above her. How fast is the distance between the girl and the balloon increasing 3 seconds later?
- 9. (10 points) Find the absolute maximum and minimum values of $f(x) = \frac{3x-4}{x^2+1}$ on the interval [-2,2].
- 10. Consider the function $f(x) = 3x^5 5x^3 + 3$.
 - (a) (6 points) Find the intervals on which f is increasing/decreasing.
 - (b) (2 points) Find the local maximum and minimum values of f.
 - (c) (6 points) Find the intervals of concavity and the inflection points.