## Final Exam Practice Problems

1. Below is the graph of a function $y=f(x)$. Fill in the chart with POS, NEG or 0 to indicate whether $f$, $f^{\prime}$ and $f^{\prime \prime}$ are positive, negative or zero at each of the indicated points $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D .


|  | $f$ | $f^{\prime}$ | $f^{\prime \prime}$ |
| :--- | :--- | :--- | :--- |
| A |  |  |  |
| B |  |  |  |
| C |  |  |  |
| D |  |  |  |

2. Find the following limits if they exist.
(a) $\lim _{x \rightarrow 0} \frac{\sqrt{5+x}-\sqrt{5}}{x}$
(b) $\lim _{x \rightarrow 1} \frac{x^{2}-1}{1-x^{4}}$
3. Compute the derivatives of the functions below.
(a) $f(x)=\left(3 x^{2}+x+1\right)^{20}$
(b) $g(x)=\ln \left(1+e^{x}\right)$
(c) $h(x)=\frac{\sqrt{1+x}}{1-x}$
(d) $j(x)=x e^{x^{2}}$
4. Compute the following indefinite integrals.
(a) $\int x^{3} e^{x^{4}} d x$
(b) $\int \frac{(3 x+4)^{2}}{x} d x$
5. Compute the following definite integrals.
(a) $\int_{1}^{2} x^{2} \sqrt{x^{3}+1} d x$
(b) $\int_{2}^{10} \frac{x}{3 x^{2}-11} d x$
6. An offshore oil well is leaking oil onto the ocean surface, forming a circular oil slick about 0.005 meters thick. If the radius of the slick is $r$ meters, then the volume of the oil spilled is $0.005 \pi r^{2}$ cubic meters. Suppose the oil is leaking at a constant rate of 20 cubic meters per hour. Find the rate at which the radius of the oil slick is increasing at a time when the radius of the oil slick is 50 meters.
7. Let

$$
f(x)=3-\frac{2}{1+x}
$$

Use the definition of the derivative as a limit to find $f^{\prime}(3)$.
8. A bacteria culture starts with 500 bacteria and grows at a rate proportional to its size. After 3 hours there are 8000 bacteria.
(a) Find an expression for the population after $t$ hours.
(b) After how many hours will the population reach 100,000 ?
9. A curve is defined by the equation

$$
x^{2}-y^{2}+4 x+8=0
$$

Give an equation for the tangent line to the curve at the point $(1,4)$.
10. Find the absolute maximum and minimum value for the function $f(x)=9 x-3 x^{2}-x^{3}$ on the interval $[-4,2]$.
11. Use a Riemann sum with 4 subintervals and left endpoints $\left(L_{4}\right)$ to estimate the integral

$$
\int_{1}^{3} 9^{x} d x
$$

