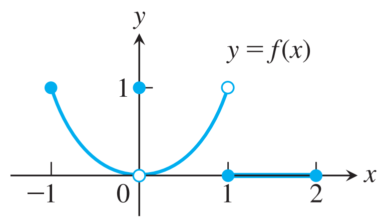
Quiz 2

Name: * Answer Key * Section: ____________

Answer all 6 questions for a total of 100 points. Write your solutions in the space provided and put a box around your final answers.

1. Use the graph below to find each of the limits. If a limit does not exist, write DNE.



(a) (4 points) $\lim_{x \to -1^+} f(x)$ = 1

(b) (4 points) $\lim_{x \to 0^+} f(x)$ = 0

(c) (4 points) $\lim_{x \to 0^-} f(x)$ = 0

(d) (4 points) $\lim_{x \to 1^+} f(x)$ = 0

(e) (4 points) $\lim_{x \to 1^-} f(x)$ = 1

(f) (4 points) $\lim_{x \to 0} f(x)$ = 0

(g) (4 points) $\lim_{x \to 1} f(x)$ D.N.E.

2. Evaluate each of the following limits. If a limit does not exist, write DNE

(a) (10 points) $\lim_{x \to -2} \frac{-2x-4}{x^3+2x^2}$

$$= \lim_{x \to -2} \frac{-2\cancel{(x+2)}}{x^2\cancel{(x+2)}}$$

$$x = -2: \quad \frac{-2}{(-2)^2} = \frac{-2}{4} = \boxed{-\frac{1}{2}}$$

(b) (10 points) $\lim_{x \to -1} \frac{\sqrt{x^2+8}-3}{x+1} \cdot \frac{\sqrt{x^2+8}+3}{\sqrt{x^2+8}+3}$

$$= \lim_{x \to -1} \frac{x^2+8-9}{(x+1)(\sqrt{x^2+8}+3)} = \lim_{x \to -1} \frac{x^2-1}{(x+1)(\sqrt{x^2+8}+3)}$$

$$= \lim_{x \to -1} \frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}(\sqrt{x^2+8}+3)}$$

$$x = -1: \quad \frac{-1-1}{\sqrt{(-1)^2+8}+3} = \frac{-2}{6} \text{ or } \boxed{-\frac{1}{3}}$$

3. There are two equivalent definitions for the derivative of a function $f(x)$ at a point a, denoted $f'(a)$.

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a} \qquad \text{(Definition 1)}$$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} \qquad \text{(Definition 2)}$$

Let $f(x) = \dfrac{x}{2-x}$.

(a) (10 points) Use Definition 1 to find $f'(4)$.

$$f'(4) = \lim_{x\to 4} \frac{f(x) - f(4)}{x - 4} = \lim_{x\to 4} \frac{\frac{x}{2-x} - \frac{4}{2-4}}{x-4}$$

$$= \lim_{x\to 4} \frac{1}{x-4}\left(\frac{x}{2-x} + \frac{2(2-x)}{2-x}\right) = \lim_{x\to 4} \frac{-x+4}{(x-4)(2-x)}$$

$$= \lim_{x\to 4} \frac{-\cancel{(x-4)}}{\cancel{(x-4)}(2-x)}$$

$$x = 4: \quad \frac{-1}{2-4} = \boxed{\frac{1}{2}}$$

(b) (6 points) Use your answer to part (a) to give an equation for the line tangent to $y = f(x)$ at the point $(4, -2)$.

$$y = f(4) + f'(4)(x-4) = -2 + \frac{1}{2}(x-4), \text{ or}$$

$$\boxed{y = \frac{1}{2}x - 4}$$

4. There are two equivalent definitions for the derivative of a function $f(x)$ at a point a, denoted $f'(a)$.

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a} \qquad \text{(Definition 1)}$$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} \qquad \text{(Definition 2)}$$

Let $f(x) = 3x^2 - 4x$.

(a) (10 points) Use Definition 2 to find $f'(2)$.

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{3(2+h)^2 - 4(2+h) - (3(2)^2 - 4(2))}{h}$$

Note: $3(2+h)^2 - 4(2+h) - (3(2)^2 - 4(2))$

$= 3(4 + 4h + h^2) - 8 - 4h - 12 + 8$

$= \cancel{12} + 12h + 3h^2 - \cancel{8} - 4h - \cancel{12} + \cancel{8}$

$= h^2 + 8h = h(h+8)$

$$f'(2) = \lim_{h\to 0} \frac{\cancel{h}(h+8)}{\cancel{h}} \xrightarrow{h=0} \boxed{8}$$

(b) (6 points) Use your answer to part (a) to give an equation for the line tangent to $y = f(x)$ at the point $(2, 4)$.

$y = f(2) + f'(2)(x-2) = 4 + 8(x-2)$, or

$$\boxed{y = 8x - 12}$$

5. (10 points) Evaluate the limit $\lim_{x \to 0} \underbrace{\frac{3}{\sqrt{3x+1}+1}}_{f(x)}$.

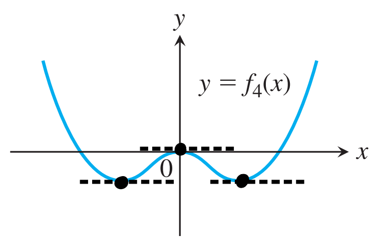
ALGEBRAIC FUNCTIONS (LIKE THIS) ARE CONTINUOUS ON THEIR DOMAIN.

SINCE 0 IS IN THE DOMAIN OF f, f IS CONTINUOUS AT 0.

THAT MEANS $\lim_{x \to 0} f(x) = f(0)$.

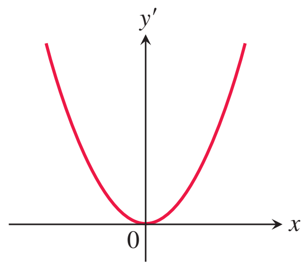
$$\lim_{x \to 0} \frac{3}{\sqrt{3x+1}+1} = \frac{3}{\sqrt{3(0)+1}+1} = \boxed{\frac{3}{2}}$$

6. (10 points) Consider the graph $y = f(x)$ below.

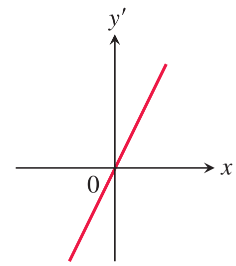


3 PLACES WHERE $f'(x) = 0$

Which one of the following graphs is the graph $y = f'(x)$? Why (briefly)?

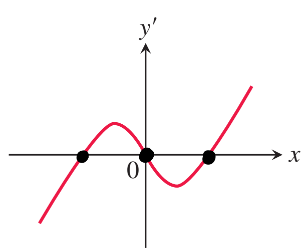


(a)

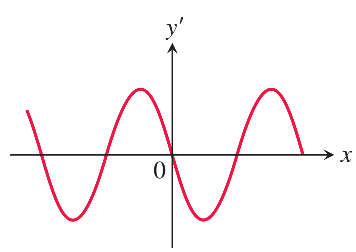


(b)

3 PLACES WHERE $f'(x) = 0$



(c) ✓



(d)