## Quiz 4

Name: $\qquad$ Section: $\qquad$

> Answer all 5 questions for a total of 100 points. Write your solutions in the space provided and put a box around your final answers. Answers can be left as logarithmic/exponential expressions, or a calculator can be used to write your answers numeriacally.

1. (25 points) The equation

$$
x^{2}-x y+y^{2}=3
$$

represents a "rotated ellipse", that is, an ellipse whose axes are not parallel to the coordinate axes. Find the points at which this ellipse crosses the $x$-axis (i.e. $x$-intercepts) and show that the tangent lines at these points are parallel (i.e. have the same slope).
2. (10 points) Find the derivative of $f(x)=\ln \left(\sqrt{\frac{3 x+2}{3 x-2}}\right)$.

Hint: the calculus will be much easier if you first apply log laws to $f(x)$.
3. (20 points) Suppose a sample of radioactive material has an initial mass of 92.3 grams and decays exponentially. If its mass 10 days later is 91.8 grams, find the half-life of the material.
4. How long does it take an investment to double if it earns $4.68 \%$ annual interest...
(a) (10 points) compounded semi-annually (twice per year)?
(b) (10 points) compounded continuously?
5. (25 points) A street light is mounted at the top of a pole 15 ft tall. A man 6 ft tall walks away from the pole with speed of $5 \mathrm{ft} / \mathrm{s}$ along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?

