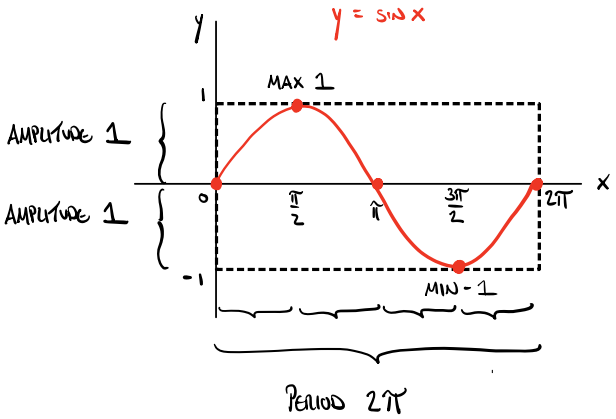
§5.3 Trig Graphs — WA 5.3 # 2, 4, 5, 6, 7

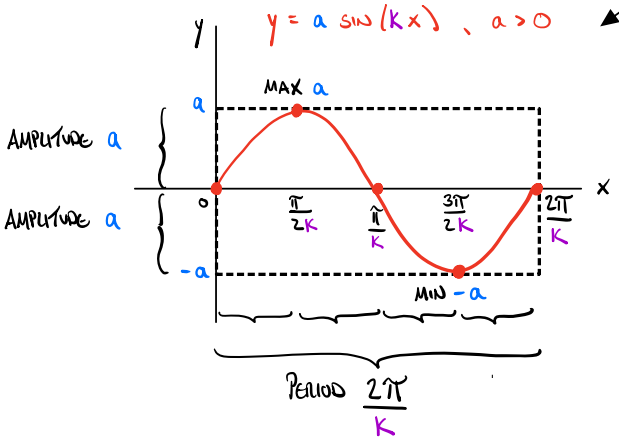


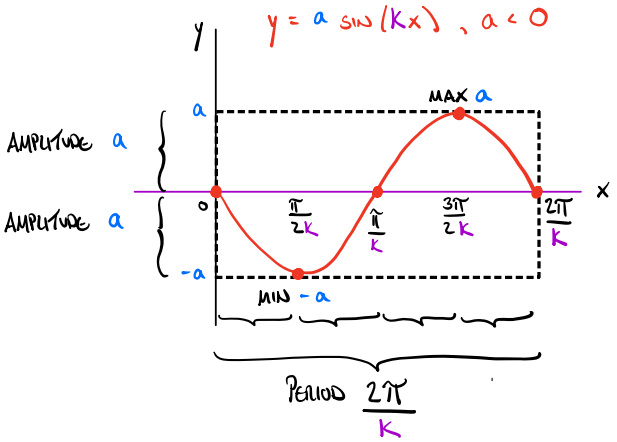
Transformations:

To graph $y = a \sin(kx)$

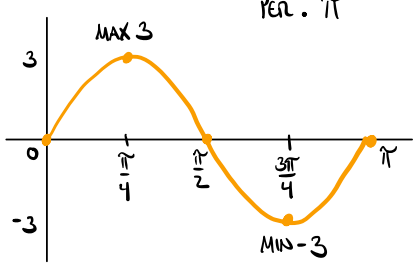
$|a|$ = Amplitude $\quad \frac{2\pi}{k}$ = Period

- multiply y-coord. by a
- divide x-coord. by k

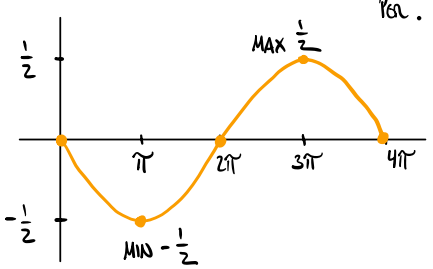




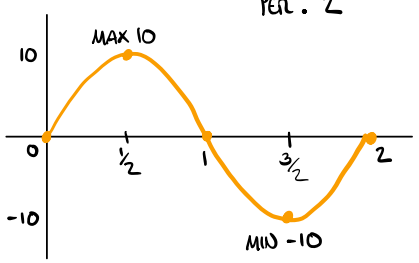
ex. $y = 3\sin(2x)$ Amp. 3, Per. π



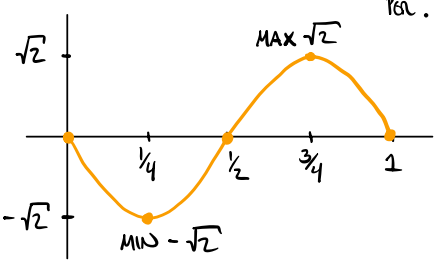
ex. $y = -\frac{1}{2}\sin(\frac{1}{2}x)$ Amp. $\frac{1}{2}$, Per. 4π



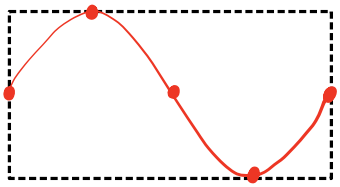
ex. $y = 10\sin(\pi x)$ Amp. 10, Per. 2



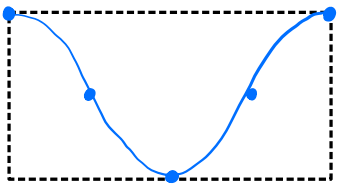
ex. $y = -\sqrt{2}\sin(2\pi x)$ Amp. $\sqrt{2}$, Per. 1



COSINE GRAPHS ARE <u>ALMOST</u> THE SAME BUT OUT OF PHASE (THE WAVE STARTS AT THE TOP INSTEAD OF MIDDLE)

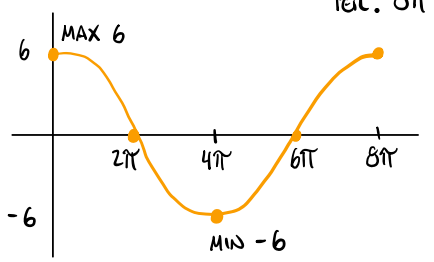


SIN

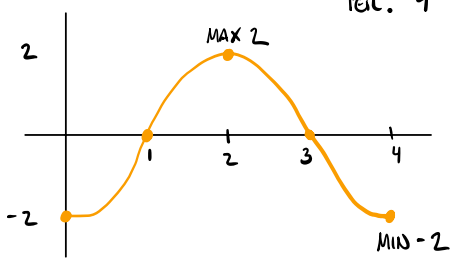


COS

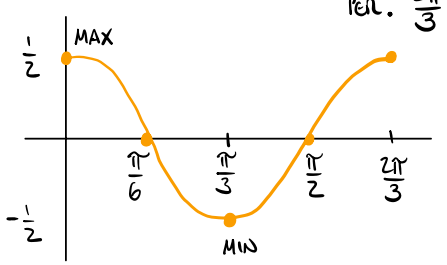
ex. $y = 6\cos\left(\frac{1}{4}x\right)$ AMP. 6 PER. 8π



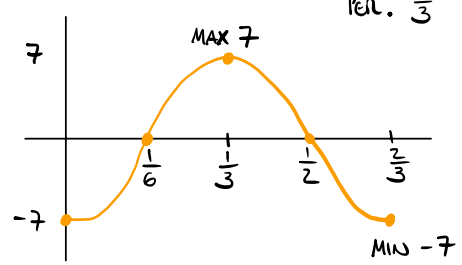
ex. $y = -2\cos\left(\frac{\pi}{2}x\right)$ AMP. 2 PER. 4



ex. $y = \frac{1}{2}\cos(3x)$ AMP. $\frac{1}{2}$ PER. $\frac{2\pi}{3}$



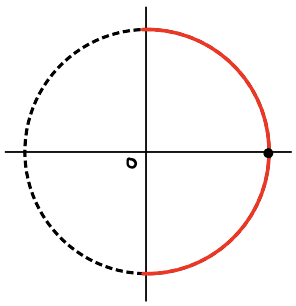
ex. $y = -7\cos(3\pi x)$ AMP. 7 PER. $\frac{2}{3}$



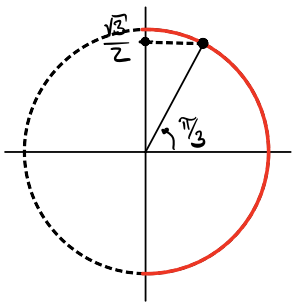
§5.5 Inverse Trig Functions

WA 5.5 #1, 2, 3, 6, 9, 10

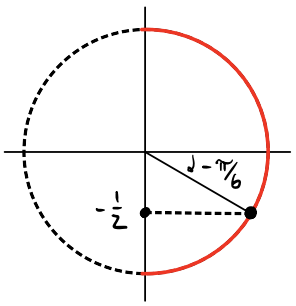
Def: $\sin^{-1}(y) = \theta \iff \sin(\theta) = y$ AND $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$



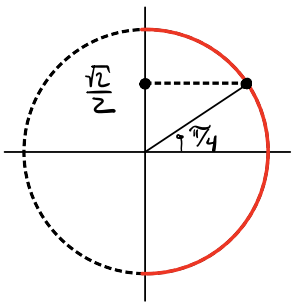
$\sin^{-1}(0) = 0$



$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

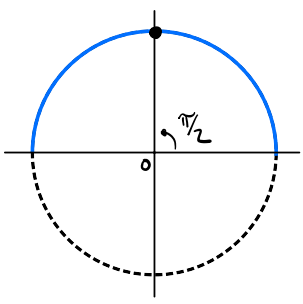


$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

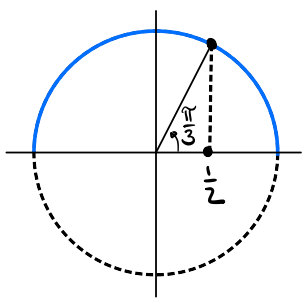


$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

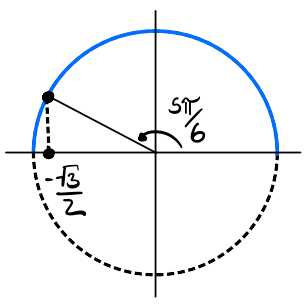
Def: $\cos^{-1}(x) = \theta \iff \cos(\theta) = x$ AND $0 \le \theta \le \pi$



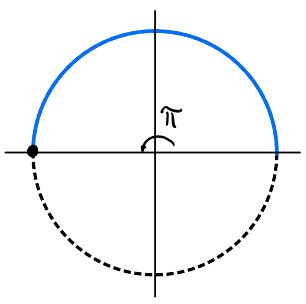
$\cos^{-1}(0) = \frac{\pi}{2}$



$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$



$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$



$\cos^{-1}(-1) = \pi$

You can use these tables to quiz yourself

y	$\sin^{-1}(y)$
-1	$-\pi/2$
$-\sqrt{3}/2$	$-\pi/3$
$-\sqrt{2}/2$	$-\pi/4$
$-1/2$	$-\pi/6$
0	0
$1/2$	$\pi/6$
$\sqrt{2}/2$	$\pi/4$
$\sqrt{3}/2$	$\pi/3$
1	$\pi/2$

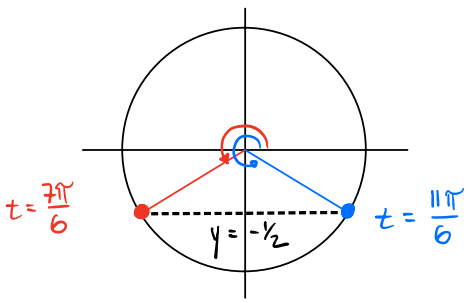
x	$\cos^{-1}(x)$
-1	π
$-\sqrt{3}/2$	$5\pi/6$
$-\sqrt{2}/2$	$3\pi/4$
$-1/2$	$2\pi/3$
0	$\pi/2$
$1/2$	$\pi/3$
$\sqrt{2}/2$	$\pi/4$
$\sqrt{3}/2$	$\pi/6$
1	0

§7.4 Solving Trig Equations

WA 7.4 # 1-6

ex. Find all solutions to: $2\sin t + 1 = 0$

$\sin t = -\frac{1}{2}$



$t = \frac{7\pi}{6} + 2\pi k \;,\; \frac{11\pi}{6} + 2\pi k$

where $k = 0, \pm 1, \pm 2, \ldots$

(1) Isolate trig expression

(2) Use unit circle to find all solutions $0 \le t \le 2\pi$

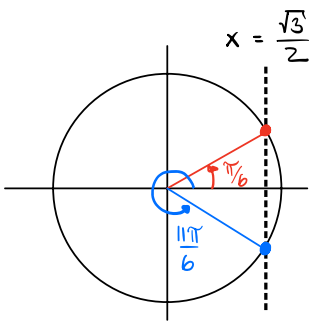
($\cos t = x$, $\sin t = y$)

(3) To find all solutions, add $2\pi k$ to all solutions from step (2), where $k = 0, \pm 1, \pm 2, \ldots$

(Note: $2\pi k$ radians $= k \times 360°$)

ex. Find all solutions to: $2\cos t = \sqrt{3}$

$\cos t = \frac{\sqrt{3}}{2}$



Solutions $0 \le t \le 2\pi$: $t = \frac{\pi}{6} \;,\; \frac{11\pi}{6}$

All solutions : $t = \underbrace{\frac{\pi}{6} + 2\pi k}_{\downarrow} \;,\; \underbrace{\frac{11\pi}{6} + 2\pi k}_{\searrow}$ where $k = 0, \pm 1, \pm 2, \ldots$

k	$\frac{\pi}{6} + 2\pi k$
⋮	⋮
-1	$-11\pi/6$
0	$\pi/6$
1	$13\pi/6$
2	$25\pi/6$
⋮	⋮

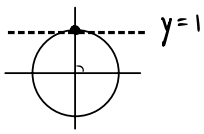
k	$\frac{11\pi}{6} + 2\pi k$
⋮	⋮
-1	$-\pi/6$
0	$11\pi/6$
1	$23\pi/6$
2	$35\pi/6$
⋮	⋮

ex. FIND ALL SOLUTIONS FOR $0 \le t \le 2\pi$

(a) $\dfrac{2}{\sin t} = 2$

$\sin(t) = 1$

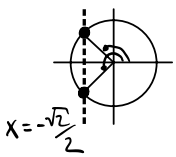
$t = \frac{\pi}{2}$



(b) $2\cos t + \sqrt{2} = 0$

$\cos(t) = -\sqrt{2}/2$

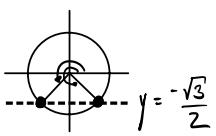
$t = \frac{3\pi}{4}, \frac{5\pi}{4}$



(c) $4\sin(t) + 2\sqrt{3} = 0$

$\sin(t) = -\sqrt{3}/2$

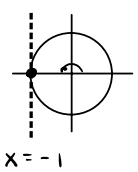
$t = \frac{4\pi}{3}, \frac{5\pi}{3}$



(d) $-3\cos(t) = 3$

$\cos(t) = -1$

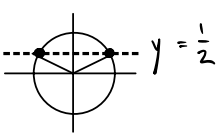
$t = \pi$



(e) $\dfrac{1}{\sin(t)} = 2$

$\sin(t) = \frac{1}{2}$

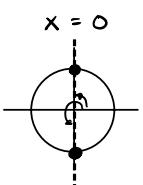
$t = \frac{\pi}{6}, \frac{5\pi}{6}$



(f) $\cos(t) = 0$

$\cos(t) = 0$

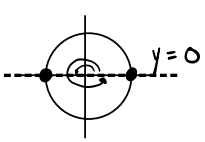
$t = \frac{\pi}{2}, \frac{3\pi}{2}$



(g) $\sin(t) = 0$

$\sin(t) = 0$

$t = 0, \pi, 2\pi$



§7.1-2 Trig Identities + Formulas

WA 7.2 # 3, 4, 7

(1) Pythagorean Identity
(2) Addition/subtraction formula for sin

Pythagorean Identity: $\sin^2 t + \cos^2 t = 1$ (True for all #'s t)

ex. Suppose $\sin t = -\frac{3}{5}$ and $\cos t > 0$. Find $\tan t$.

$\sin^2 t + \cos^2 t = 1$

$\left(-\frac{3}{5}\right)^2 + \cos^2 t = 1$

$\cos^2 t = 1 - \left(-\frac{3}{5}\right)^2 = \frac{16}{25}$

$\cos t = \oplus\sqrt{\frac{16}{25}} = \frac{4}{5}$

$\tan t = \frac{\sin t}{\cos t} = \frac{-3/5}{4/5} = \boxed{-\frac{3}{4}}$

ex. Suppose $\cos t = \frac{12}{13}$ & $\sin t < 0$. Find $\tan t$.

$\sin^2 t + \left(\frac{12}{13}\right)^2 = 1$

$\sin^2 t = 1 - \left(\frac{12}{13}\right)^2 = \frac{25}{169}$

$\sin t = \ominus\sqrt{\frac{25}{169}} = -\frac{5}{13}$

$\tan t = \frac{\sin t}{\cos t} = \frac{-5/13}{12/13} = \boxed{-\frac{5}{12}}$

ex. Suppose $\sin t = \frac{4}{5}$ & $\cos t < 0$. Find $\tan t$.

$\left(\frac{4}{5}\right)^2 + \cos^2 t = 1$

$\cos^2 t = 1 - \left(\frac{4}{5}\right)^2 = \frac{9}{25}$

$\cos t = \ominus\sqrt{\frac{9}{25}} = -\frac{3}{5}$

$\tan t = \frac{\sin t}{\cos t} = \frac{4/5}{-3/5} = \boxed{-\frac{4}{3}}$

Addition Formula for sin: $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

Subtraction Formula for sin: $\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$

Note: $\frac{\pi}{6} = \frac{2\pi}{12}$, $\frac{\pi}{4} = \frac{3\pi}{12}$, $\frac{\pi}{3} = \frac{4\pi}{12}$

ex. $\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{3\pi}{12} - \frac{2\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

ex. $\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

ex. $\sin\left(-\frac{7\pi}{12}\right) = -\sin\left(\frac{7\pi}{12}\right)$

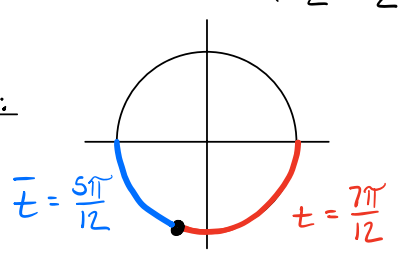
Recall: $\sin(-t) = -\sin(t)$, $\cos(-t) = \cos(t)$

$= -\sin\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right) = -\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$= -\left(\sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}\right)$

$= -\left(\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}\right) = \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$

Alt:



$-\frac{7\pi}{12}$ terminates in QIII with reference # $\bar{t} = \frac{5\pi}{12}$

$\therefore \sin\left(-\frac{7\pi}{12}\right) = -\sin\left(\frac{5\pi}{12}\right) = -\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)$ ✓