Name: * Answer Key *

Each question is worth 5 points. Show your work in the space provided and write your final answer *neatly* on the answer line. Good luck!

1. Simplify $\left(\frac{1}{3}+\frac{3}{4}\right)\left(1-\frac{1}{7}\right)$.

$$\left(\frac{1}{3}\cdot\frac{4}{4}+\frac{3}{4}\cdot\frac{3}{3}\right)\left(1\cdot\frac{7}{7}-\frac{1}{7}\right)$$

$$=\left(\frac{4}{12}+\frac{9}{12}\right)\left(\frac{6}{7}\right)=\frac{13}{12}\cdot\frac{6}{7}$$

$$=\frac{13\cdot 6}{2\cdot 6\cdot 7}=\frac{13}{14}$$

1. $\frac{13}{14}$

2. Simplify $\left(\frac{15}{2+\frac{1}{2}}\right)^2$.

$$\left(\frac{15}{2\cdot\frac{2}{2}+\frac{1}{2}}\right)^2=\left(\frac{15}{\frac{4}{2}+\frac{1}{2}}\right)^2=\left(\frac{15}{\frac{5}{2}}\right)^2$$

$$=\left(15\cdot\frac{2}{5}\right)^2=\left(3\cdot 5\cdot\frac{2}{5}\right)^2=6^2=36$$

2. 36

3. Simplify $\left(\frac{-4x^5y^{-7}}{3x^{-3}y}\right)^{-1}$ and eliminate any negative exponents.

$$= \left(\frac{-4x^5x^3}{3yy^7}\right)^{-1} = \left(\frac{-4x^8}{3y^8}\right)^{-1} = \frac{3y^8}{-4x^8}$$

3. $-\frac{3y^8}{4x^8}$

4. Perform the multiplication $9x^{3/2}\left(7\sqrt{x} - \frac{6}{\sqrt{x}}\right)$ and simplify.

$$9x^{3/2}\left(7x^{1/2} - 6x^{-1/2}\right) = \left(9x^{3/2}\right)\left(7x^{1/2}\right) - \left(9x^{3/2}\right)\left(6x^{-1/2}\right)$$

$$= 63x^{\frac{3}{2}+\frac{1}{2}} - 54x^{\frac{3}{2}-\frac{1}{2}}$$

$$= 63x^2 - 54x$$

4. $63x^2 - 54x$

5. Evaluate $\left(\frac{27}{8}\right)^{-2/3}$.

$$\left(\left(\frac{27}{8}\right)^{1/3}\right)^{2}\Bigg)^{-1} = \left(\left(\frac{3}{2}\right)^{2}\right)^{-1} = \left(\frac{9}{4}\right)^{-1} = \frac{4}{9}$$

5. $\frac{4}{9}$

6. Factor $-2x^4 + 28x^3 - 64x^2$ completely.

$$-2x^2\left(x^2 - 14x + 32\right)$$

GREATEST COMMON FACTOR

6. $-2x^2(x^2 - 14x + 32)$

7. Perform the division $\dfrac{9x^2-1}{x^2+2x+1} \div \dfrac{3x^2-2x-1}{x^2-1}$ and simplify.

$$\frac{(3x+1)(3x-1)}{(x+1)^2} \div \frac{(3x+1)(x-1)}{(x+1)(x-1)}$$

$$\frac{(3x+1)(3x-1)}{(x+1)^2} \times \frac{(x+1)(x-1)}{(3x+1)(x-1)} = \frac{3x-1}{x+1}$$

(KEEP, CHANGE, FLIP)

7. $\dfrac{3x-1}{x+1}$

8. Perform the addition/subtraction $\dfrac{1}{2} - \dfrac{2}{x+2} + \dfrac{4}{(x+2)^2}$ and simplify.

LCD $= 2(x+2)^2$

$$\frac{1}{2} \cdot \frac{(x+2)^2}{(x+2)^2} - \frac{2}{x+2} \cdot \frac{2(x+2)}{2(x+2)} + \frac{4}{(x+2)^2} \cdot \frac{2}{2}$$

$$= \frac{(x+2)^2 - 4(x+2) + 8}{2(x+2)^2} = \frac{x^2+4x+4-4x-8+8}{2(x+2)^2}$$

$$= \frac{x^2+4}{2(x+2)^2}$$

8. $\dfrac{x^2+4}{2(x+2)^2}$

9. Find all real solutions of the equation $x^2 - 8x = -13$.

COMPLETING THE SQUARE:

$$x^2 - 8x + 16 = -13 + 16$$

HALF, SQR

$$(x-4)^2 = 3$$

$$x - 4 = \pm\sqrt{3}$$

$$x = 4 \pm \sqrt{3}$$

9. $4 \pm \sqrt{3}$

10. Find all real solutions of the equation $\frac{9}{x} + 4 = \frac{7}{x-2}$

MULTIPLY BOTH SIDES OF EQUATION (ALL TERMS) BY LCD: $x(x-2)$

$$\frac{9}{x} \cdot x(x-2) + 4 \cdot x(x-2) = \frac{7}{x-2} \cdot x(x-2)$$

$$9(x-2) + 4x(x-2) = 7x$$

$$9x - 18 + 4x^2 - 8x = 7x$$

$$4x^2 - 6x - 18 = 0$$

$$2(2x^2 - 3x - 9) = 0$$

$$2(2x+3)(x-3) = 0$$

(1) (2)

(1) $2x + 3 = 0$

$x = -3/2$

(2) $x - 3 = 0$

$x = 3$

10. $-3/2, 3$

11. Solve the equation $P = \dfrac{nRT}{V}$ for V.

$V \cdot P = \dfrac{nRT}{V} \cdot V \quad \Rightarrow \quad \dfrac{VP}{P} = \dfrac{nRT}{P}$

$\Rightarrow \quad V = \dfrac{nRT}{P}$

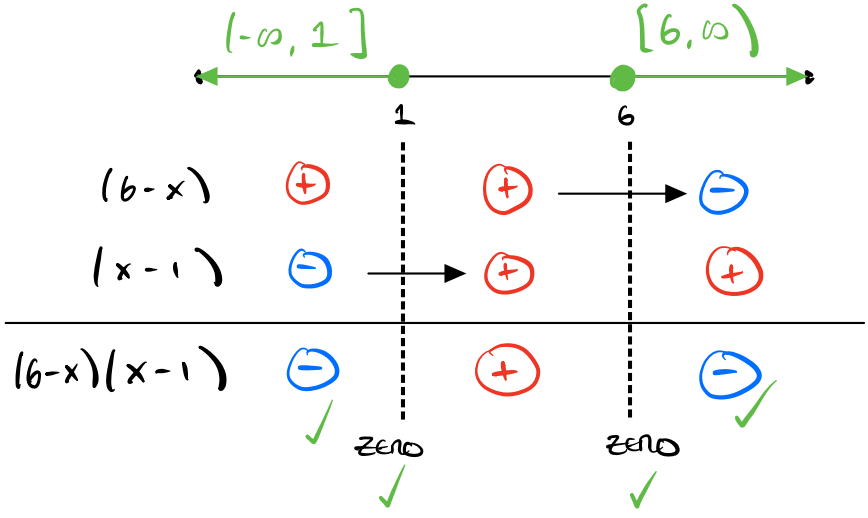
11. $V = \dfrac{nRT}{P}$

12. Solve the nonlinear inequality $6(x-1) \leq x(x-1)$. Express your answer using interval notation.

$6(x-1) - x(x-1) \leq 0$

$(6-x)(x-1) \leq 0 \quad \Rightarrow$ zeros $x = 1, \; x = 6$

(NEGATIVE OR ZERO)



12. $(-\infty, 1] \cup [6, \infty)$

13. Find all real solutions of the equation $\sqrt{8-x}+2=x-4$.

ISOLATE $\sqrt{\ }$ EXPRESSION BEFORE SQUARING BOTH SIDES.

$\left(\sqrt{8-x}\right)^2 = (x-6)^2$

WARNING: WHEN SQUARING BOTH SIDES WE MAY INTRODUCE FALSE SOLUTIONS. CHECK YOUR ANSWERS!

$8-x = x^2-12x+36$

$0 = x^2-11x+28$

$0 = (x-7)(x-4)$

$x-7=0$ or $x-4=0$

$x=7$ $\quad x=4$

CHECK: $x=7$: $\sqrt{8-7}+2 = 7-4$ ✓

$x=4$: $\sqrt{8-4}+2 = 4-4$ ⊗

13. 7

14. Find the center and radius of the circle with the equation $x^2+y^2+4x=9+12y$.

$x^2+4x+4 + y^2-12y+36 = 9+4+36$

$(x+2)^2 + (y-6)^2 = 49$

$(x+2)^2 + (y-6)^2 = 7^2$

14. CENTER $(-2,6)$, $r=7$

15. Give an equation for the line that passes through the points $(-1,-2)$ and $(4,3)$.

$x_1, y_1 \quad x_2, y_2$

$$\text{SLOPE } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-2)}{4-(-1)} = \frac{5}{5} = 1$$

POINT-SLOPE FORMULA FOR LINE THRU (a,b) WITH SLOPE m:

$$y - b = m(x-a)$$

$m = 1$, $(a,b) = (3,4)$ $\longrightarrow$ $y - 4 = x - 3$

$m = 1$, $(a,b) = (-1,-2)$ $\longrightarrow$ $y + 2 = x + 1$

$$\Big\} \; y = x - 1$$

15. $y = x - 1$

16. Evaluate and simplify $\dfrac{f(a+h) - f(a)}{h}$ when $f(x) = 4x^2 - 3x + 9$.

$f(\;) = 4(\;)^2 - 3(\;) + 9$

$f(a+h) = 4(a+h)^2 - 3(a+h) + 9 = 4a^2 + 8ah + 4h^2 - 3a - 3h + 9$

$$\frac{f(a+h) - f(a)}{h} = \frac{4a^2 + 8ah + 4h^2 - 3a - 3h + 9 - (4a^2 - 3a + 9)}{h}$$

$$= \frac{4a^2 + 8ah + 4h^2 - 3a - 3h + 9 - 4a^2 + 3a - 9}{h}$$

$$= \frac{h(8a + 4h - 3)}{h} = 8a + 4h - 3$$

16. $8a + 4h - 3$

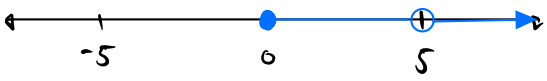
17. Find the domain of the function $g(x) = \dfrac{\sqrt{x}}{x^2 - 25}$. Express your answer using interval notation.

$\sqrt{x}$: $x \geq 0$

$\div (x^2 - 25)$: $x^2 - 25 \neq 0$

$(x+5)(x-5) \neq 0$

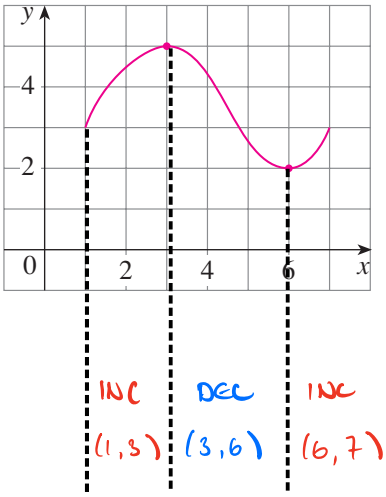
$x \neq -5$, $x \neq 5$



$[0,5) \cup (5,\infty)$

17. $[0,5) \cup (5,\infty)$

18. The graph $y = f(x)$ is shown below. Use interval notation to state the interval(s) on which f is increasing and find the average rate of change in f from 3 to 7.



18. $(1,3) \cup (6,7)$

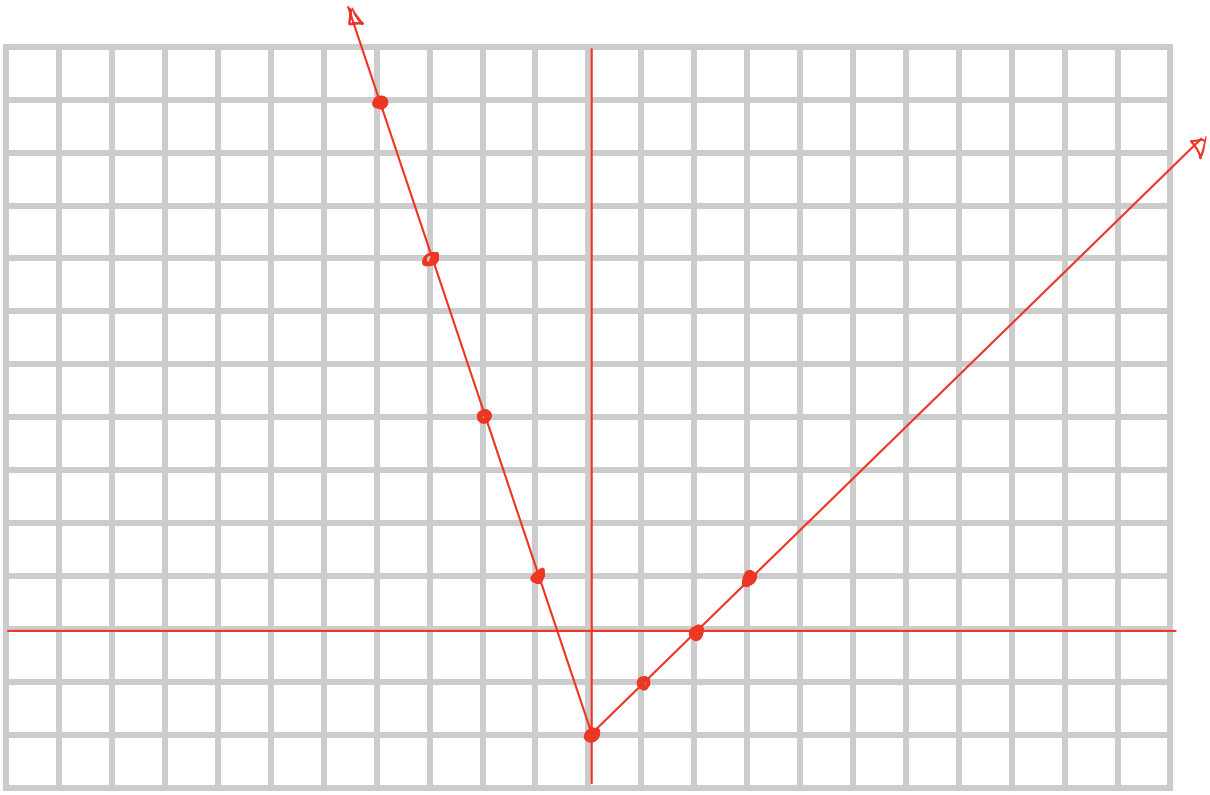
19. Sketch the graph $y = |2x| - x - 2$ by first completing the table of values below and then plotting points.

x	y
-4	10
-3	7
-2	4
-1	1
0	-2
1	-1
2	0
3	1

$= |2x| - x - 2$

$x - 2$ IF $x \geq 0$

$-3x - 2$ IF $x < 0$



20. Sketch the graph of the following piecewise defined function.

$$f(x) = \begin{cases} \frac{1}{3}x + 1 & \text{if } x < 3 \\ 4 & \text{if } x = 3 \\ 1 & \text{if } x > 3 \end{cases}$$