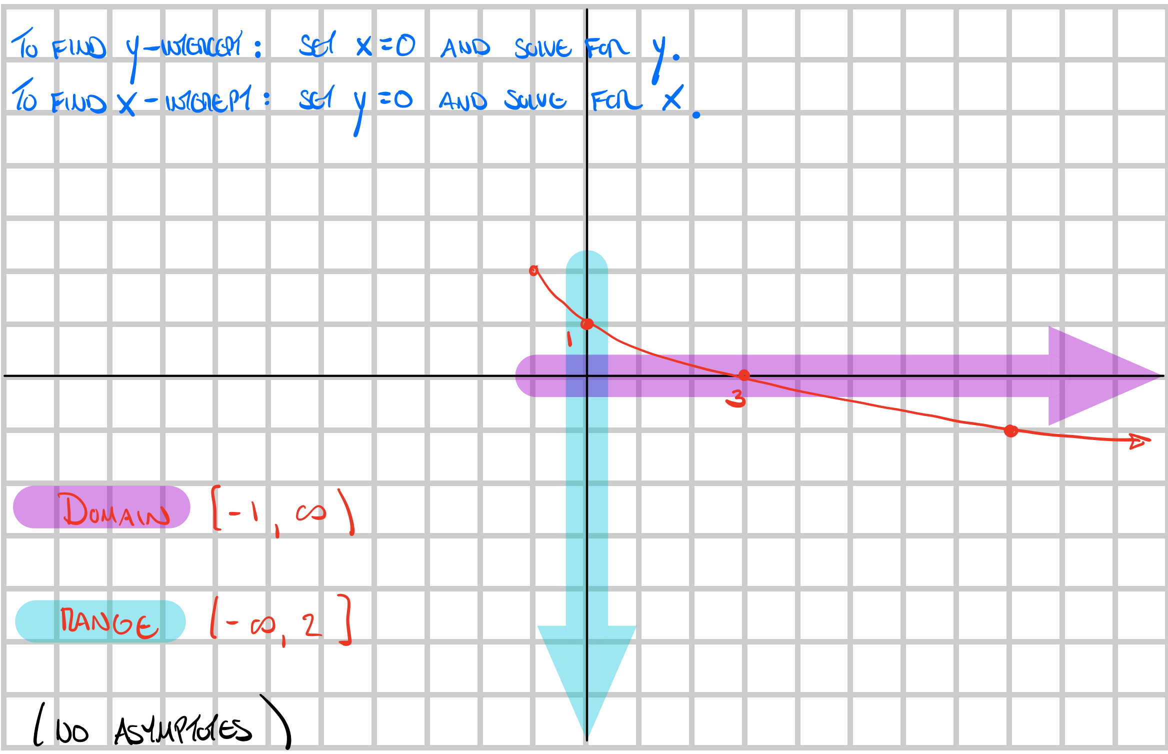
Name: * ANSWER KEY *

Each question is worth 5 points. Show your work in the space provided and **put a box around your final answer**. Answers should be simplified, but can include logarithmic and/or exponential expressions. Good luck!

1. Sketch the graph of $f(x) = 2 - \sqrt{x+1}$. Label any/all x-intercepts, y-intercepts, horizontal asymptotes, and vertical asymptotes. State the domain and range using interval notation.



2. Suppose $f(x) = 2x^2 + x$ and $g(x) = 3 - x$. Find $f(g(x))$ and $g(f(x))$.

WHAT DOES f DO? $f(\quad) = 2(\quad)^2 + (\quad)$

$f(g(x)) = f(3-x) = 2(3-x)^2 + (3-x)$

$= 2(x^2 - 6x + 9) + (3-x) = 2x^2 - 12x + 18 + 3 - x$

$= \boxed{2x^2 - 13x + 21}$

WHAT DOES g DO? $g(\quad) = 3 - (\quad)$

$g(f(x)) = g(2x^2 + x) = 3 - (2x^2 + x) = \boxed{-2x^2 - x + 3}$

3. Let f be the one-to-one function $f(x) = \dfrac{3}{x-4}$. Find $f^{-1}(x)$.

SET $y = \dfrac{3}{x-4}$. SOLVE FOR x TO FIND $f^{-1}(y)$, THEN SWITCH x & y.

$$(x-4)\, y = \frac{3}{\cancel{x-4}}\, \cancel{(x-4)} \Rightarrow xy - 4y = 3$$

$$xy = 3 + 4y$$

$$x = \frac{3+4y}{y} \rightsquigarrow \boxed{f^{-1}(x) = \frac{3+4x}{x}}$$

$$\left(= \frac{3}{x} + 4 \right)$$

4. Use the following table to evaluate $g(f^{-1}(2))$.

x	0	1	2	3	4	5
$f(x)$	1	4	3	0	2	5
$g(x)$	4	2	3	1	5	0

DEFINITION OF INVERSE FUNCTION: $f^{-1}(2) = 4$ BECAUSE $f(4) = 2$.

THEN $g(f^{-1}(2)) = g(4) = \boxed{5}$

5. Consider the quadratic function $q(x) = x^2 - 12x + 40$. Use "completing the square" to write $q(x)$ in standard form. Then determine the maximum/minimum value of $q(x)$ and state whether it is a maximum or a minimum.

$$q(x) = (\underbrace{x^2 - 12x}_{\text{HALF, SQUARE}}) + 40 = (x^2 - 12x + 36) + 40 - 36 = \boxed{(x-6)^2 + 4}$$

$$\geq 0 \quad + 4$$

$$q(x) \geq 4$$

IN STANDARD FORM, $q(x) = a(x-h)^2 + k$

HAS A

- MINIMUM VALUE k IF $a > 0$.
- MAXIMUM VALUE k IF $a < 0$.

$\boxed{q(x) \text{ HAS MINIMUM VALUE } 4}$

6. Find the maximum/minimum value of $f(x) = -\dfrac{x^2}{3} + 2x + 7$ and state whether it is a maximum or minimum.

A QUADRATIC FUNCTION $f(x) = ax^2 + bx + c$ HAS

- MINIMUM VALUE $f\left(-\frac{b}{2a}\right)$ IF $a > 0$
- MAXIMUM VALUE $f\left(-\frac{b}{2a}\right)$ IF $a < 0$.

WE HAVE $f(x) = \frac{-1}{3}x^2 + 2x + 7 \Rightarrow a = \frac{-1}{3},\ b = 2,\ c = 7.$

$f\left(-\frac{b}{2a}\right) = f\left(\frac{-\cancel{2}}{\cancel{2}(-\frac{1}{3})}\right) = f\left(\frac{-1}{-\frac{1}{3}}\right) = f(3) = -\frac{1}{3}(3)^2 + 2(3) + 7 = -3 + 6 + 7 = \boxed{10 \text{ MAXIMUM}}$

7. Let $y = -7x^5 - x^4 + 5x + 2$. Describe the end behavior of f by filling in the blanks:

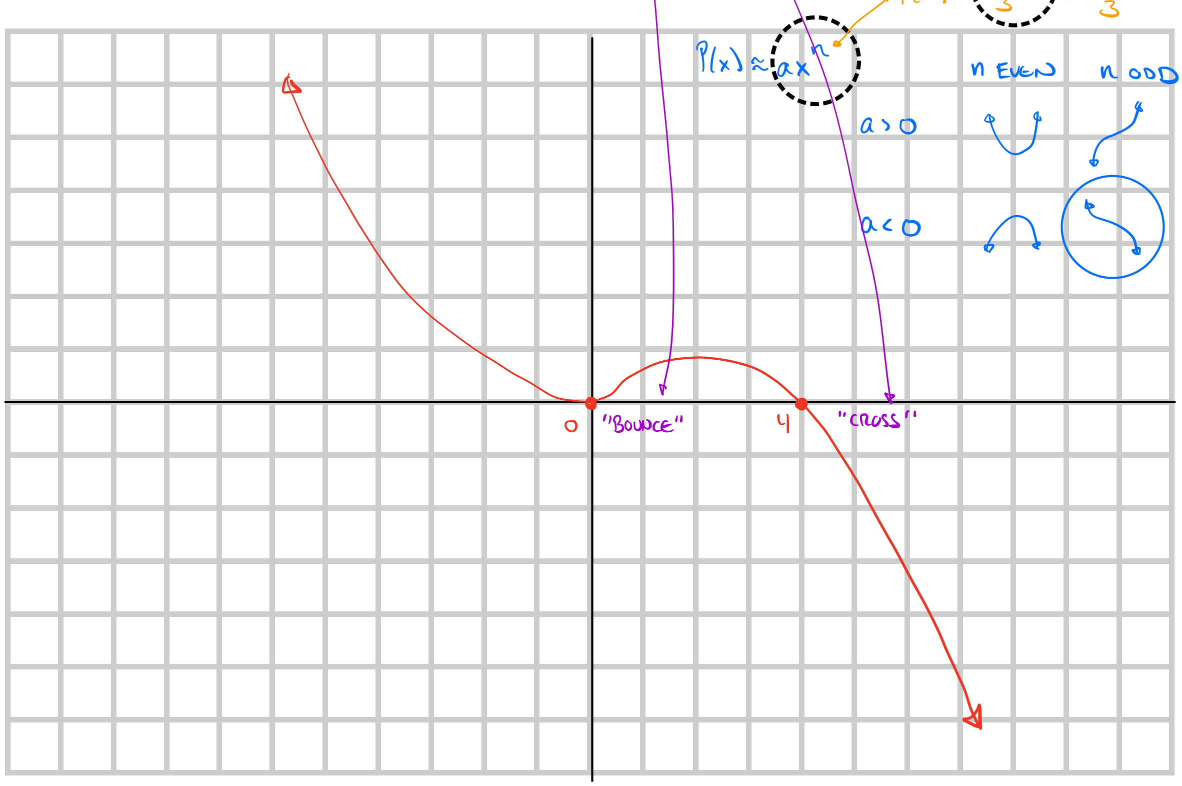
As $x \to -\infty$, $y \to$ $\underline{\ \infty\ }$.

As $x \to \infty$, $y \to$ $\underline{\ -\infty\ }$.

EVEN ODD

8. Sketch the graph of the polynomial $P(x) = -\frac{2}{3}x^{2}(x-4)^{1}$. Label any/all x-intercepts, y-intercepts, horizontal asymptotes, and vertical asymptotes. Make sure your graph exhibits the proper end behavior and correctly shows where $P(x)$ is positive/negative.

$P(x) = -\frac{2}{3}x^3 + \frac{8}{3}x^2$



9. Solve: $7^{1-2x} = 7^{3x-5}$.

$$\log_7\left(7^{1-2x}\right) = \log_7\left(7^{3x-5}\right)$$

$$1-2x = 3x-5$$
$$6 = 5x$$
$$\boxed{x = \frac{6}{5}}$$

10. Solve $8e^{x/3} = 40$.

$$\frac{1}{8}\cdot 8e^{x/3} = \frac{1}{8}\cdot 40$$

$$e^{x/3} = 5 \qquad \ln\left(e^{x/3}\right) = \ln 5$$

$$\frac{x}{3} = \ln 5 \qquad \frac{x}{3}\underbrace{\ln(e)}_{1} = \ln 5$$

$$\boxed{x = 3\ln 5}$$

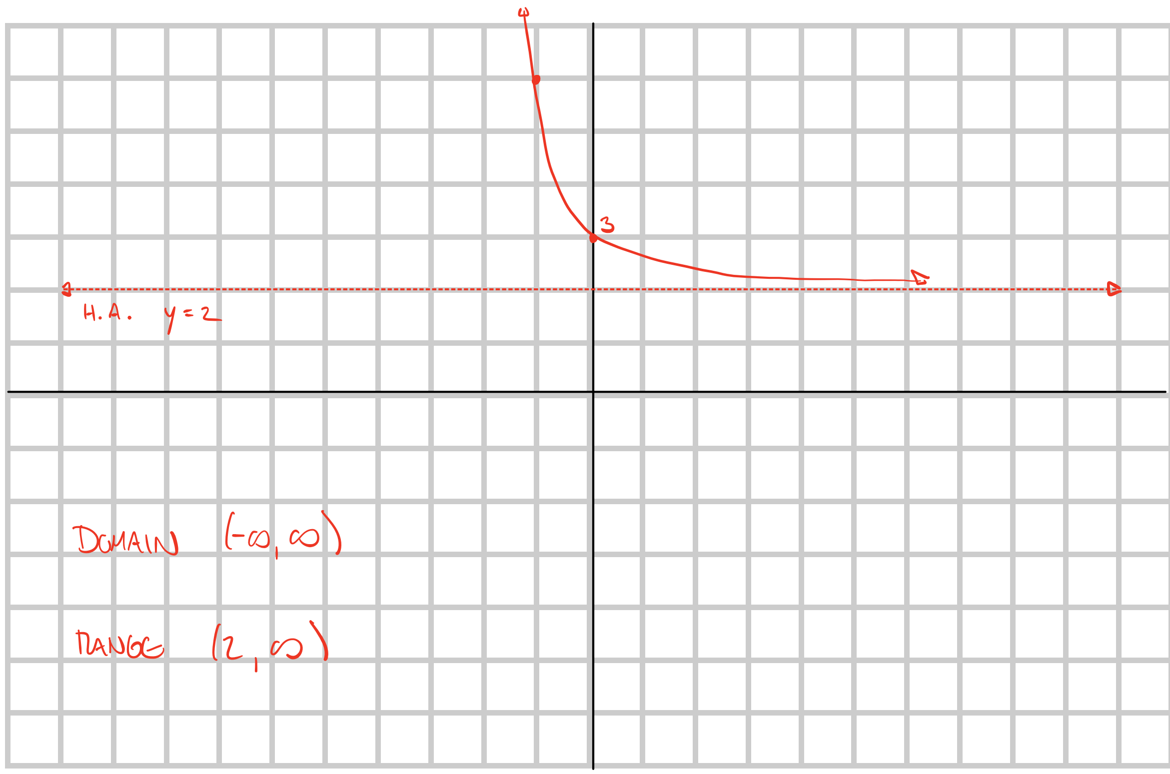
11. Solve: $\dfrac{50}{1+e^{-x}} = 4$.

$$(1+e^{-x})\frac{50}{1+e^{-x}} = 4(1+e^{-x})$$

$$50 = 4 + 4e^{-x}$$
$$46 = 4e^{-x}$$
$$\frac{46}{4} = e^{-x}$$
$$\ln\left(\frac{46}{4}\right) = -x$$

$$\boxed{x = -\ln\left(\frac{23}{2}\right)}$$

12. Sketch the graph of $f(x) = 2 + 4^{-x}$. Label any/all x-intercepts, y-intercepts, horizontal asymptotes, and vertical asymptotes. State the domain and range using interval notation.



13. Evaluate $\log_2\left(\frac{1}{32}\right)$. $= \boxed{-5}$

BECAUSE $2^5 = 32$, SO $2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

$\left(\log_a x = y \iff a^y = x \right)$

14. Use log laws to evaluate $3\ln(2) + 2\ln(3) - \ln(72)$.

$= \ln(2^3) + \ln(3^2) - \ln(72)$

$= \ln(2^3 \cdot 3^2) - \ln(72)$

$= \ln\left(\frac{2^3 \cdot 3^2}{72}\right) = \ln\left(\frac{8 \cdot 9}{72}\right)$

$= \ln(1)$

$= \boxed{0}$

Log Laws

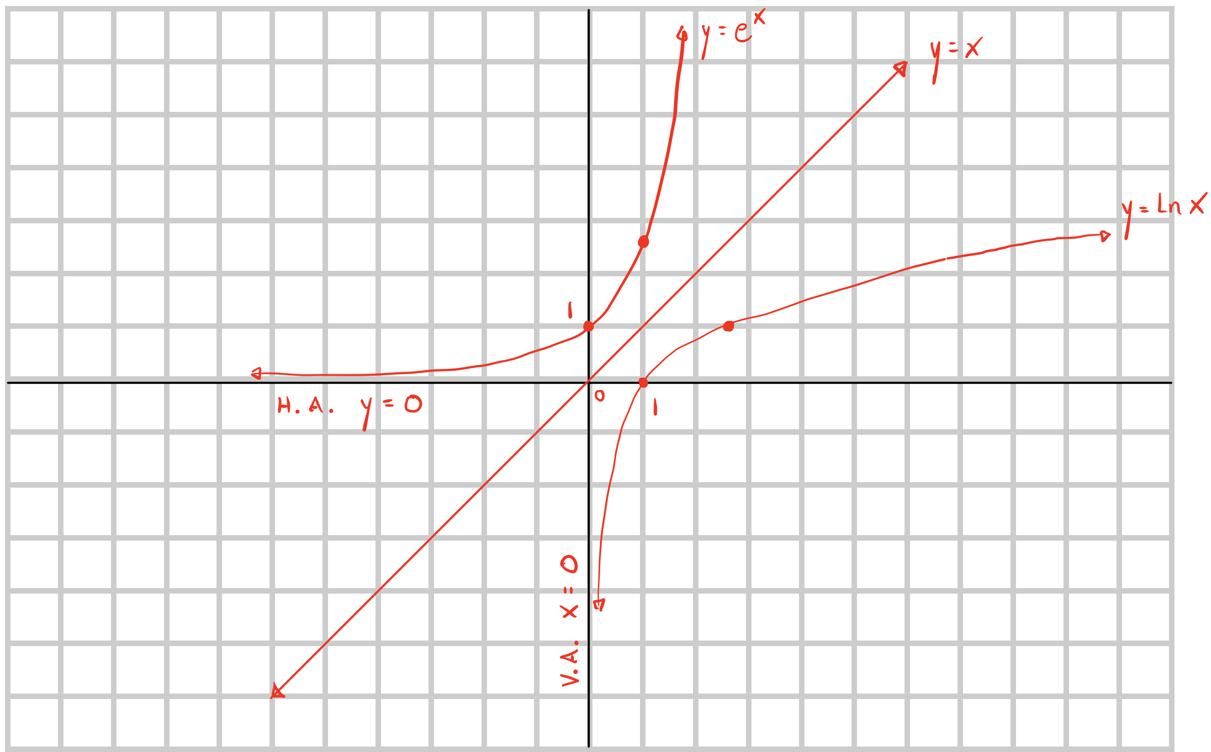
(1) $\log(AB) = \log A + \log B$

(2) $\log\left(\frac{A}{B}\right) = \log A - \log B$

(3) $\log(A^c) = c \log A$

15. Sketch the graphs of all three of the following functions on the same set of axes below. Label any/all x-intercepts, y-intercepts, horizontal asymptotes, and vertical asymptotes.

$$f(x) = x$$
$$g(x) = e^x$$
$$h(x) = \ln(x)$$



16. Let $f(x) = \ln(1 - 6x)$. State the domain of f using interval notation.

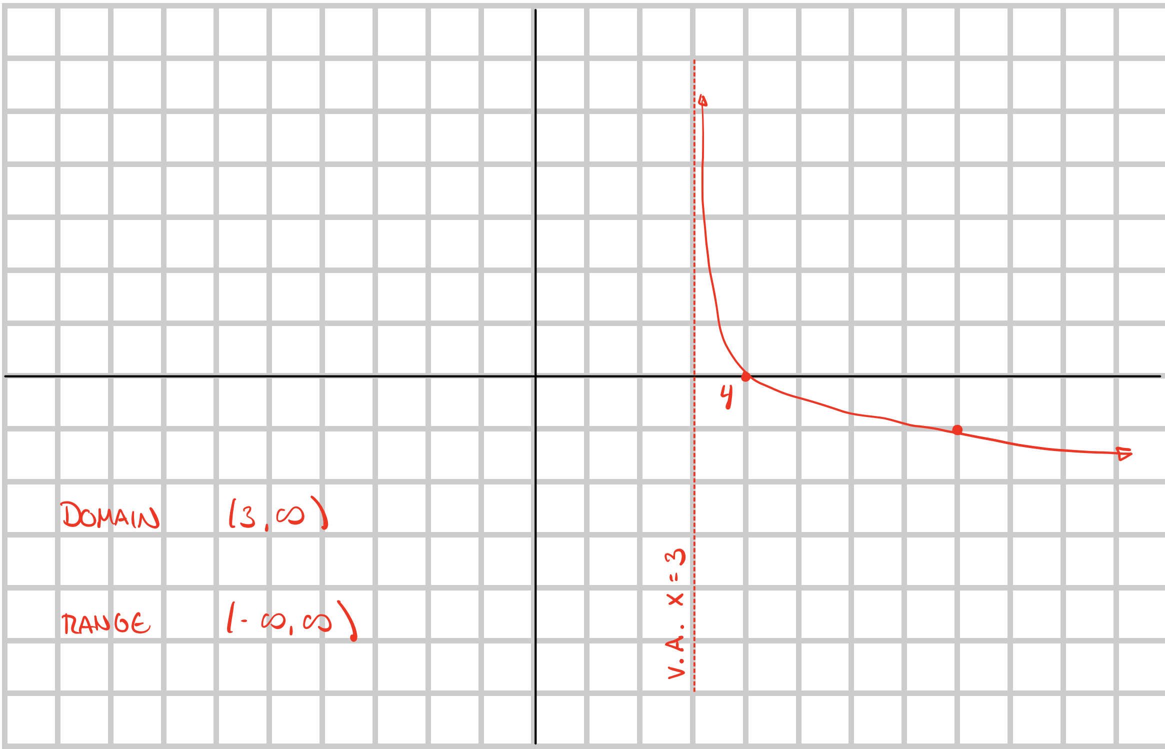
$1 - 6x > 0$

$1 > 6x$

$\frac{1}{6} > x$ or $x < \frac{1}{6}$

$\boxed{(-\infty, \frac{1}{6})}$

17. Sketch the graph of $f(x) = -\log_5(x-3)$. Label any/all x-intercepts, y-intercepts, horizontal asymptotes, and vertical asymptotes. State the domain and range using interval notation.



18. Solve: $2\ln(x) = \ln(2) + \ln(3x-4)$.

$\ln(x^2) = \ln(2(3x-4))$

$\cancel{e}^{\cancel{\ln}(x^2)} = \cancel{e}^{\cancel{\ln}(2(3x-4))}$

$x^2 = 6x - 8$

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$\boxed{x = 2, 4}$

CHECK: $x = 2$: $2\ln 2 = \ln 2 + \ln 2$ ✓

$x = 4$: $2\ln 4 = \ln 2 + \ln 8$

$\ln(4^2) = \ln(2 \cdot 8)$ ✓

19. Solve: $\log_5(x+1) - \log_5(x-1) = 2$.

$$\log_5\left(\frac{x+1}{x-1}\right) = 2$$

$$\cancel{5}^{\cancel{\log_5}\left(\frac{x+1}{x-1}\right)} = 5^2$$

$$\frac{x+1}{x-1} = 25$$

$$\cancel{(x-1)}\frac{x+1}{\cancel{x-1}} = 25\,(x-1)$$

$$x+1 = 25x - 25$$

$$26 = 24x$$

$$x = \frac{26}{24} = \boxed{\frac{13}{12}}$$

CHECK:

$$\log_5\left(\frac{25}{12}\right) - \log_5\left(\frac{1}{12}\right) = 2$$

$$\log_5\left(\frac{25/12}{1/12}\right) = 2$$

$$\log_5(25) = 2 \checkmark$$

20. Solve: $\log(x+2) + \log(x-1) = 1$.

$$\log\left((x+2)(x-1)\right) = 1$$

$$10^{\log(x^2+x-2)} = 10^1$$

$$x^2 + x - 2 = 10$$

$$x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

$$x = -4, 3$$

FALSE SOLUTION

$$\boxed{x = 3}$$

CHECK:

$x=-4$: $\underbrace{\log(-2)}\ + \underbrace{\log(-5)} = 1$

UNDEFINED :(

DOMAIN OF $\log x$ IS $(0,\infty)$

- YOU CAN ONLY TAKE log OF POSITIVE NUMBERS.

$x=3$ $\log(5) + \log(2) = 1$

$$\log(10) = 1 \checkmark$$