You have 2hr 15min. Answer each non-graph question neatly on the line provided.

Name: * ANSWER KEY * ID: ________________

1. (4 points) Simplify $\left(\frac{2a^{-1}b}{a^4b^{-2}}\right)^3$ and eliminate negative exponents.

$$= \left(\frac{2b^3}{a^5}\right)^3 = \frac{2^3(b^3)^3}{(a^5)^3} = \frac{8b^9}{a^{15}}$$

1. $\frac{8b^9}{a^{15}}$

2. (4 points) Simplify $64^{-\frac{1}{2}}$ completely.

$$= \frac{1}{64^{1/2}} = \frac{1}{\sqrt{64}} = \frac{1}{8}$$

2. $\frac{1}{8}$

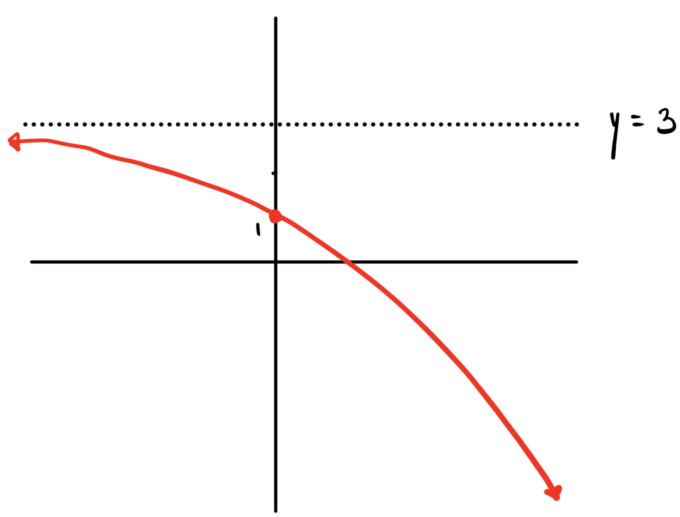
3. (4 points) Factor $2x^2 + 4x - 96$ completely.

$2(x^2 + 2x - 48)$

$2(x+8)(x-6)$

3. $2(x+8)(x-6)$

4. (4 points) Sketch the graph of $f(x) = -2^{x+1} + 3$. Label all asymptotes on your graph for full credit.



5. (4 points) Perform the addition $\frac{4}{x^2} + \frac{9}{x^2+6}$ and simplify as one reduced fraction.

$LCD = x^2(x^2+6)$

$$\frac{4(x^2+6) + 9x^2}{x^2(x^2+6)} = \frac{13x^2+24}{x^2(x^2+6)}$$

5. $\frac{13x^2+24}{x^2(x^2+6)}$

6. (4 points) Perform the multiplication $\frac{4x^2}{x^2-81} \cdot \frac{2x+18}{16x}$ and simplify as one reduced fraction.

$$\frac{\cancel{(4x)}x}{(\cancel{x+9})(x-9)} \quad \frac{\cancel{2}(\cancel{x+9})}{\cancel{(4x)}2\cdot\cancel{2}} = \frac{x}{2(x-9)}$$

6. $\frac{x}{2(x-9)}$

7. (4 points) Find all solutions x to $\frac{2}{x+2} - \frac{4}{x^2} = 0$.

$$x^2(x+2)\left(\frac{2}{x+2} - \frac{4}{x^2}\right) = (0)\, x^2(x+2$$

$$2x^2 - 4(x+2) = 0$$

$$2x^2 - 4x - 8 = 0$$

$$2(x^2 - 2x - 4) = 0$$

$$x^2 - 2x - 4 = 0$$

$$(x-1)^2 - 5 = 0$$

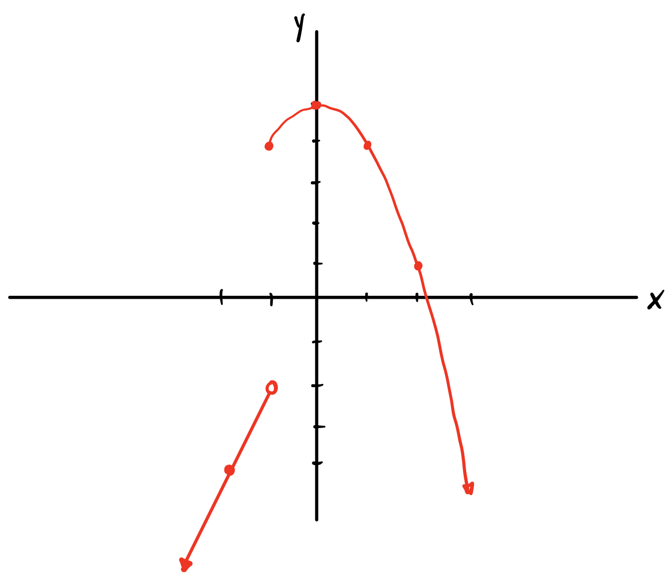
$$(x-1)^2 = 5$$

$$x - 1 = \pm\sqrt{5}$$

$$x = 1 \pm \sqrt{5}$$

7. $1 \pm \sqrt{5}$

8. (4 points) Sketch the graph of the piecewise function $f(x) = \begin{cases} 2x & \text{if } x < -1 \\ 5 - x^2 & \text{if } x \geq -1 \end{cases}$



9. (4 points) Find the radius of the circle with equation $x^2 + y^2 - \frac{1}{2}x + \frac{1}{2}y = \frac{1}{8}$.

$$x^2 - \frac{1}{2}x + \frac{1}{16} + y^2 + \frac{1}{2}y + \frac{1}{16} = \frac{1}{8} + \frac{1}{16} + \frac{1}{16}$$

$$\left(x - \frac{1}{4}\right)^2 + \left(y + \frac{1}{4}\right)^2 = \frac{1}{4}$$

$$r^2 = \frac{1}{4} \Rightarrow r = \frac{1}{2}$$

9. $\frac{1}{2}$

10. (4 points) Find an equation of the line that passes through the points $(-1, -2)$ and $(7, -6)$.

$x_1 \; y_1 \qquad x_2 \; y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - (-2)}{7 - (-1)} = \frac{-6 + 2}{7 + 1} = \frac{-4}{8} = -\frac{1}{2}$$

$y + 2 = -\frac{1}{2}(x + 1)$

or

10. $y = -\frac{1}{2}x - \frac{5}{2}$

11. (4 points) Evaluate $\sin\left(-\frac{5\pi}{12}\right)$.

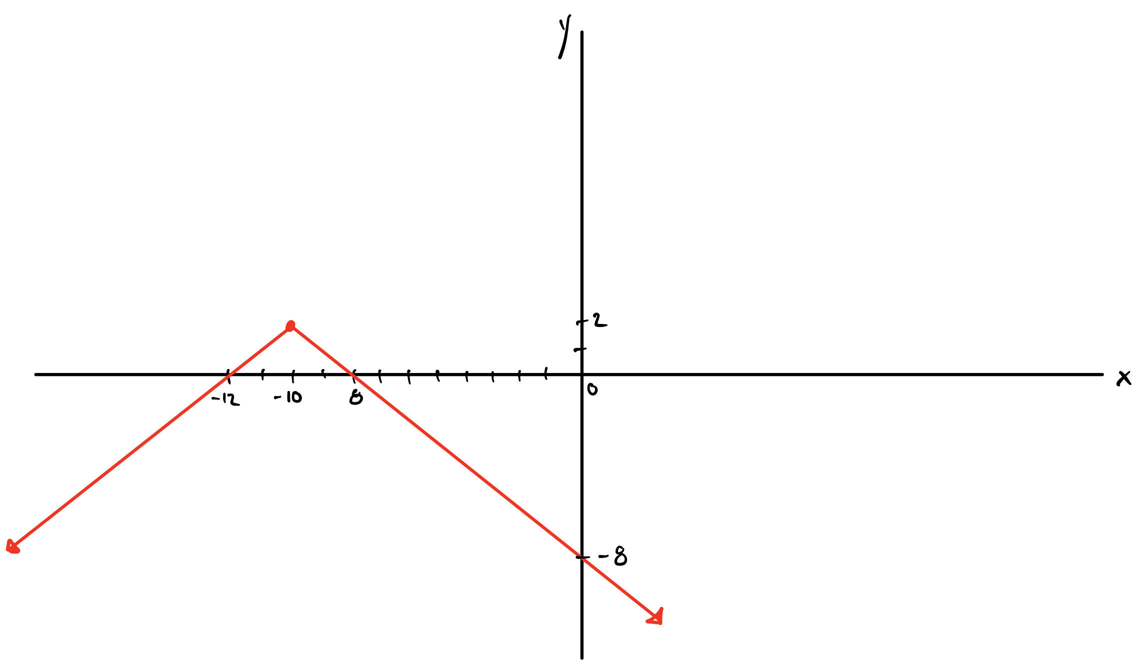
$$\sin\left(-\frac{5\pi}{12}\right) = -\sin\left(\frac{5\pi}{12}\right) = -\sin\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right) = -\sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

$$= -\left(\sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3}\right)$$

$$= -\frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$

11. $\dfrac{-\sqrt{2} - \sqrt{6}}{4}$

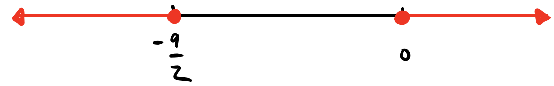
12. (4 points) Sketch the graph of $y = 2 - |x + 10|$.

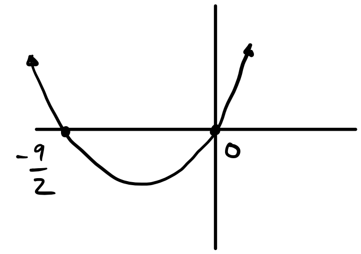


13. (4 points) Solve $x(2x+9) \geq 0$ for x. Express the solution using interval notation.

ZEROS: $x=0$ $\quad 2x+9=0$

$x = -\frac{9}{2}$





		$-\frac{9}{2}$		0	
x	$-$		$-$		$+$
$2x+9$	$-$		$+$		$+$
$x(2x+9)$	$+$		$-$		$+$

13. $(-\infty, -\frac{9}{2}] \cup [0, \infty)$

14. (4 points)

FIND $\cos(t)$ IF $\sin(t) = \frac{3}{5}$ AND $\tan(t) < 0$.

$\sin^2 t + \cos^2 t = 1$

$\cos^2 t = 1 - \sin^2 t$

$\cos t = \pm\sqrt{1 - \sin^2 t} \longrightarrow \cos(t) = -\sqrt{1 - \left(\frac{3}{5}\right)^2} \longrightarrow -\frac{4}{5}$

SINCE $\tan(t) = \dfrac{\sin(t)}{\cos(t)}$ IS NEGATIVE, AND $\sin(t)$ IS POSITIVE, $\cos(t)$ MUST BE NEGATIVE.

14. $-\frac{4}{5}$

15. (4 points) Find the average rate of change of $f(t) = t - \frac{2}{t}$ between $t = \frac{-1}{4}$ and $t = \frac{1}{2}$.

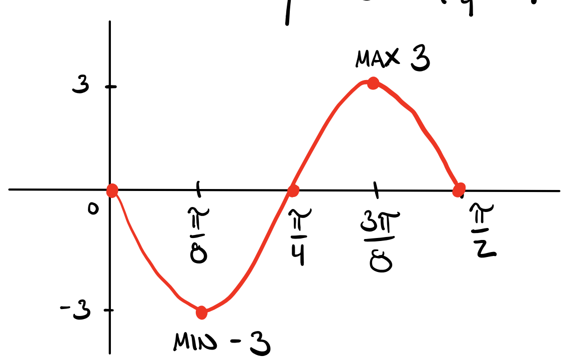
$$\frac{f\left(\frac{1}{2}\right) - f\left(-\frac{1}{4}\right)}{\frac{1}{2} - \left(-\frac{1}{4}\right)} = \frac{\left(\frac{1}{2} - 4\right) - \left(-\frac{1}{4} + 8\right)}{3/4} = \frac{4}{3}\left(\frac{1}{2} - 4 + \frac{1}{4} - 8\right)$$

$$= \frac{4}{3}\left(\frac{3}{4} - 12\right) = 1 - 16 = -15$$

15. -15

16. (4 points) SKETCH THE GRAPH OF ONE COMPLETE PERIOD OF THE FUNCTION $y = -3\sin\left(\frac{1}{4}x\right)$. LABEL ALL INTERCEPTS, MAXIMUMS, AND MINIMUMS.

$y = -3\sin\left(\frac{1}{4}x\right)$



- SIN CURVE REFLECTED ACROSS X-AXIS
- AMPLITUDE 3
- PERIOD $\frac{2\pi}{4} = \frac{\pi}{2}$

17. (4 points) Find the inverse function of $f(x) = \frac{1}{x+9}$.

$$x = \frac{1}{y+9} \quad \Rightarrow \quad y+9 = \frac{1}{x}$$

$$y = \frac{1}{x} - 9$$

17. $f^{-1}(x) = \frac{1}{x} - 9$

18. (4 points) Evaluate and simplify $f(10+h) - f(10)$ when $f(x) = -2x^2 + x + 5$.

$$f(10+h) = -2(10+h)^2 + (10+h) + 5$$
$$= -2h^2 - 39h - 185$$

$$f(10) = -2(10)^2 + (10) + 5$$
$$= -200 + 15 = -185$$

$$f(10+h) - f(10) = -2h^2 - 39h - \cancel{185} - (-\cancel{185}) = -2h^2 - 39h$$

18. $-2h^2 - 39h$

19. (4 points) Use the table

x	1	2	3	4	5	6
f(x)	2	3	5	1	6	3
g(x)	3	4	1	5	2	6

to evaluate $g(f(g^{-1}(2)))$ $\cancel{f(g(f^{-1}(2)))}$

SINCE $g(5) = 2$, $g^{-1}(2) = 5$.

THEN $g(f(g^{-1}(2))) = g(f(5)) = g(6) = 6$

19. 6

20. (4 points) In 2010 the deer population in a Pennsylvania county was 20000. In 2014 the deer population in the county had grown to 31000. Assuming the deer population in the county is growing exponentially, approximate the county's deer population in 2023. (You may leave log or e in your answer.)

$$n(t) = n_0 e^{rt}$$

$$n(0) = n_0 = 20{,}000$$

$$n(4) = 20{,}000\, e^{r4} = 31{,}000$$

$$e^{r4} = \frac{31}{20}$$

$$e^r = \left(\frac{31}{20}\right)^{1/4} \Rightarrow r = \frac{1}{4}\ln\left(\frac{31}{20}\right)$$

20. $20{,}000 \left(\frac{31}{20}\right)^{\frac{13}{4}}$

or $20{,}000\, e^{\frac{1}{4}\ln\left(\frac{31}{20}\right)\cdot 13}$

$$n(13) = 20{,}000 \left(\frac{31}{20}\right)^{\frac{13}{4}} \quad \text{or} \quad 20{,}000\, e^{\frac{1}{4}\ln\left(\frac{31}{20}\right)\cdot 13}$$

21. (4 points) Evaluate $\log_{36}(\frac{1}{6})$.

$= \log_{36}(1) - \log_{36}(6)$

$= 0 - \frac{1}{2} = -\frac{1}{2}$

21. $-\frac{1}{2}$

22. (4 points) Solve $2\log x = \log 2 + \log(4x - 6)$ for x.

$\log(x^2) = \log(8x - 12) \xrightarrow{x=6:} \log 36 = \log(48-12)$ ✓

$\xrightarrow{x=2} \log 4 = \log(16-12)$ ✓

$x^2 = 8x - 12$

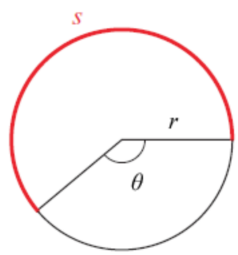
$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

$x = 6 \qquad x = 2$

22. $x = 2, 6$

23. (4 points) Find the length s of the circular arc



when $r = 8$ and $\theta = 120°$.

$S = r\theta$ WHEN θ IS MEASURED IN RADIANS.

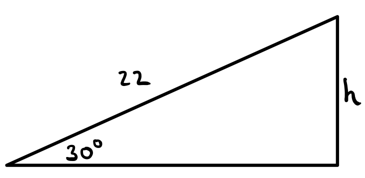
$120 \text{ DEG} \cdot \frac{\pi \text{ RAD}}{180 \text{ DEG}} = \frac{2\pi}{3}$

23. $\frac{32\pi}{3}$

NOTE: THE GIVEN θ IS NOT THE ANGLE CORRESPONDING TO S.

WE NEED $\theta = 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$ $\Rightarrow S = 8 \cdot \frac{4\pi}{3} = \frac{32\pi}{3}$

24. (4 points) A 22-ft ladder leans against a building so that the angle between the ground and the ladder is 30°. How high does the ladder reach on the building?



$\sin 30° = \frac{h}{22}$

$\Rightarrow h = 22 \sin 30°$

$= 22\left(\frac{1}{2}\right) = 11$

24. 11 ft

25. (4 points) Evaluate $\sin^{-1}(\cos(\frac{7\pi}{6}))$.

$\cos\left(\frac{7\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

REFERENCE #

25. $-\frac{\pi}{3}$

$\sin^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

BECAUSE $\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ AND $-\frac{\pi}{2} \le -\frac{\pi}{3} \le \frac{\pi}{2}$