

§1.3 SUBSETS

Def: Let A, B be sets. A is a **SUBSET** of B , $A \subseteq B$,
 IF EVERY ELEMENT OF A IS ALSO AN ELEMENT OF B .
 A is **NOT A SUBSET** of B , $A \not\subseteq B$, IF THERE IS
 AN ELEMENT OF A THAT IS NOT ALSO AN ELEMENT OF B .

NOTE: ONE &
 ONLY ONE MUST
 BE TRUE!

eg. $\{1, 2\} \subseteq \{1, 2, 3\}$

$\{1, 2\} \not\subseteq \{1, \{2, 3\}\}$

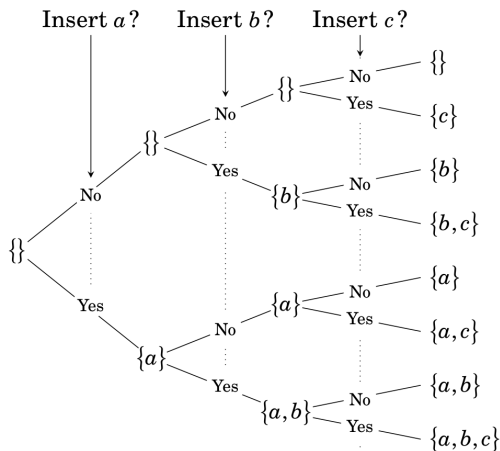
$\{2, 3\} \not\subseteq \{1, \{2, 3\}\}$, $\{2, 3\} \in \{1, \{2, 3\}\}$

3. $\{1\} \in \{1, \{1\}\}$ $\{1\}$ is the second element listed in $\{1, \{1\}\}$
4. $\{1\} \subseteq \{1, \{1\}\}$ make subset $\{1\}$ by selecting 1 from $\{1, \{1\}\}$
5. $\{\{1\}\} \in \{1, \{1\}\}$ because $\{1, \{1\}\}$ contains only 1 and $\{1\}$, and not $\{\{1\}\}$
6. $\{\{1\}\} \subseteq \{1, \{1\}\}$ make subset $\{\{1\}\}$ by selecting $\{1\}$ from $\{1, \{1\}\}$

$\{\{2, 3\}\} \subseteq \{1, \{2, 3\}\}$

FACT: GIVEN ANY SET B , (i) $\emptyset \subseteq B$
 (ii) $B \subseteq B$

EX. HOW MANY SUBSETS ARE THERE OF $\{a, b, c\}$?



FACT: IF A IS A FINITE
 SET WITH n ELEMENTS
 THEN A HAS 2^n
 SUBSETS, i.e.
 $2^{|A|}$.

Figure 1.3. A "tree" for listing subsets

B. Write out the following sets by listing their elements between braces.

9. $\{X : X \subseteq \{3, 2, a\} \text{ and } |X| = 2\}$

11. $\{X : X \subseteq \{3, 2, a\} \text{ and } |X| = 4\}$

10. $\{X \subseteq \mathbb{N} : |X| \leq 1\}$

12. $\{X : X \subseteq \{3, 2, a\} \text{ and } |X| = 1\}$

C. Decide if the following statements are true or false. Explain.

13. $\mathbb{R}^3 \subseteq \mathbb{R}^3$

15. $\{(x, y) \in \mathbb{R}^2 : x - 1 = 0\} \subseteq \{(x, y) \in \mathbb{R}^2 : x^2 - x = 0\}$

14. $\mathbb{R}^2 \subseteq \mathbb{R}^3$

16. $\{(x, y) \in \mathbb{R}^2 : x^2 - x = 0\} \subseteq \{(x, y) \in \mathbb{R}^2 : x - 1 = 0\}$

§ 1.4 Power Sets

Definition 1.4 If A is a set, the **power set** of A is another set, denoted as $\mathcal{P}(A)$ and defined to be the set of all subsets of A . In symbols, $\mathcal{P}(A) = \{X : X \subseteq A\}$.

ex. List elements of $\mathcal{P}(\{a\} \times \{1, 2\})$

12. $\{X \in \mathcal{P}(\{1, 2, 3\}) : 2 \in X\}$

Fact 1.4 If A is a finite set, then $|\mathcal{P}(A)| = 2^{|A|}$.

ex. What is $\mathcal{P}(\mathbb{N})$?

ex. What is $\mathcal{P}(\mathbb{R}^2)$?

ex. What is $\mathcal{P}(\{(a, b) \in \mathbb{R} : a, b \in \mathbb{R}, a < b\})$?