# CH 3: COUNTING

HOW MANY?

e.g. IF 5 PEOPLE GATHER TOGETHER & EVERY PERSON SHAKES HANDS WITH EVERY ONE ELSE, HOW MANY HANDSHAKES TAKE PLACE?

## §3.1 LISTS

**Def:** A **LIST** IS AN ORDERED SEQUENCE OF OBJECTS

$(a,b,c) \neq (c,b,a)$

REPEATS ALLOWED: $(a,a,a,b)$ — 4 ENTRIES, i.e. **LENGTH** 4.

EQUAL LISTS HAVE SAME ENTRIES IN SAME ORDER.
IN PARTICULAR, EQUAL LISTS HAVE THE SAME LENGTH (DIFF. LENGTH ⟹ DIFF. LISTS)

ex. THE OUTCOME OF FLIPPING A COIN 5 TIMES CAN BE CONSIDERED A LIST $(T,T,H,T,H)$ → OR, AS STRING: TTHTH

**Def:** A **STRING** OF LENGTH $n$ IS A LIST OF $n$ SYMBOLS WITH PARENTHESES & COMMAS SUPPRESSED.

## §3.2 THE MULTIPLICATION PRINCIPLE

ex SANDWICH SHOP HAS
BREADS $B = \{$WHITE, WHEAT, SOURDOUGH$\}$
MEATS $M = \{$TURKEY, BEEF, HAM$\}$
CHEESE $C = \{$CHEDDAR, AMERICAN$\}$.

TO ORDER A SANDWICH, CHOOSE 1 BREAD, 1 MEAT, & 1 CHEESE.
HOW MANY DIFFERENT SANDWICHES CAN YOU ORDER?

~ HOW MANY LISTS $(x,y,z)$ CAN BE MADE WITH $x \in B$, $y \in M$, $z \in C$?

**MULTIPLICATION PRINCIPLE:** SUPPOSE WHEN MAKING A LIST OF LENGTH $n$,

$$(\square, \square, \square, \dots, \square).$$

\# POSSIBLE CHOICES: $a_1$, $a_2$, $a_3$, …, $a_n$

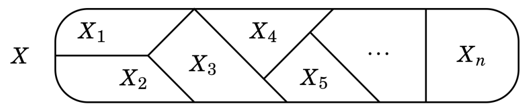
THEN TOTAL NUMBER OF DIFFERENT LISTS MADE THIS WAY IS $a_1 \times a_2 \times \dots \times a_n$.

**3.** How many lists of length 3 can be made from the symbols *A, B, C, D, E, F* if...

**(a)** ... repetition is allowed.
**(b)** ... repetition is not allowed.
**(c)** ... repetition is not allowed and the list must contain the letter $A$.
**(d)** ... repetition is allowed and the list must contain the letter $A$.

## §3.3 THE ADDITION & SUBTRACTION PRINCIPLES

**Fact 3.2 (Addition Principle)**
Suppose a finite set $X$ can be decomposed as a union $X = X_1 \cup X_2 \cup \cdots \cup X_n$, where $X_i \cap X_j = \emptyset$ whenever $i \neq j$. Then $|X| = |X_1| + |X_2| + \cdots + |X_n|$.




**3.** Five cards are dealt off of a standard 52-card deck and lined up in a row. How many such lineups are there in which all 5 cards are of the same color (i.e., all black or all red)?

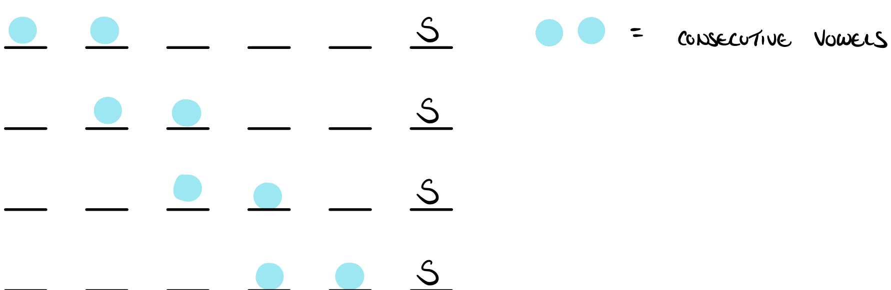
ORDER MATTERS!

**Fact 3.3 (Subtraction Principle)**
If $X$ is a subset of a finite set $U$, then $|\overline{X}| = |U| - |X|$.
In other words, if $X \subseteq U$ then $|U - X| = |U| - |X|$.

**5.** How many integers between 1 and 9999 have no repeated digits? How many have at least one repeated digit?

THINK $\{$1-DIGIT #'s$\} \cup \{$2-DIGIT #'s$\} \cup \{$3-DIGIT #'s$\} \cup \{$4-DIGIT #'s$\}$

**9.** Consider lists of length 6 made from the letters *A, B, C, D, E, F, G, H*. How many such lists are possible if repetition is not allowed and the list contains two consecutive vowels?



## §3.4 Factorials & Permutations

n-factorial

**Definition 3.1** If $n$ is a non-negative integer, then $n!$ is the number of lists of length $n$ that can be made from $n$ symbols, without repetition. Thus $0! = 1$ and $1! = 1$. If $n > 1$, then $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$.

**Note:** $n! = n(n-1)! = n(n-1)(n-2)! = \ldots$

**Def:**

Given a set $X$,
A **permutation** of $X$ is a non-repetitive list made from all elements of $X$.
A **k-permutation** of $X$ is a non-repetitive list made from $k$ elements of $X$.

Thus # permutations of a set with $n$ elements is $n!$

**Fact 3.4** A **k-permutation** of an $n$-element set is a non-repetitive length-$k$ list made from elements of the set. Informally we think of a $k$-permutation as an arrangement of $k$ of the set's elements in a row.

The number of $k$-permutations of an $n$-element set is denoted $P(n,k)$, and

$$P(n,k) = \underbrace{n(n-1)(n-2)\cdots(n-k+1)}_{k \text{ factors}}.$$

If $0 \le k \le n$, then $P(n,k) = n(n-1)(n-2)\cdots(n-k+1) = \frac{n!}{(n-k)!}$.

**15.** In a club of 15 people, we need to choose a president, vice-president, secretary, and treasurer. In how many ways can this be done?