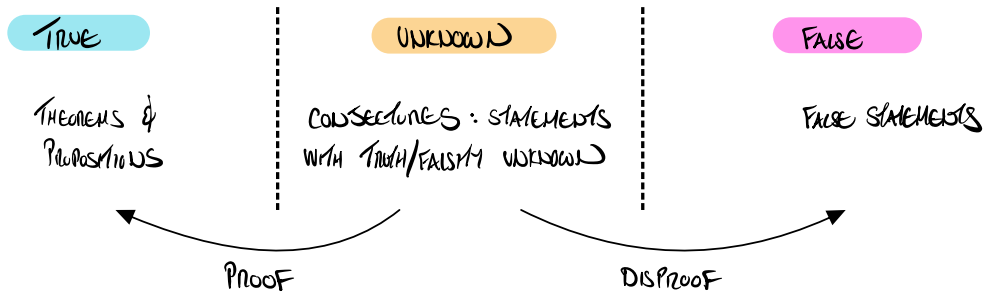


Ch 9. Disproof

CATEGORIZING STATEMENTS:



To disprove P , we must prove $\sim P$ USING DIRECT, CONTRAPOSITIVE, CONTRADICTION.

§9.1 COUNTEREXAMPLES

UNIVERSAL STATEMENTS

To prove $Q: \forall x \in S, P(x)$, we prove $\sim Q: \sim(\forall x \in S, P(x)) = \exists x \in S, \sim P(x)$.

THIS ONLY REQUIRES US TO FIND ONE SUCH x , CALLED A COUNTEREXAMPLE.

ex. PROVE OR DISPROVE: FOR ALL POSITIVE REAL NUMBERS a & b , $a+b < 2\sqrt{ab}$.

(UNIVERSAL STATEMENT: DISPROVE WITH COUNTEREXAMPLE.)

THE UNIVERSAL STATEMENT IS FALSE.

AS A COUNTEREXAMPLE, TAKE $a=15$ & $b=2$.

THEN $a+b=17$ AND $2\sqrt{ab}=12$,

SO IT IS NOT TRUE THAT $a+b < 2\sqrt{ab}$ FOR ALL POSITIVE REAL NUMBERS a & b . ■

NOTE: YOU ARE RESPONSIBLE FOR SHOWING WHY YOUR COUNTEREXAMPLE IS INDEED A COUNTEREXAMPLE!

CONDITIONAL STATEMENTS

RECALL: $P(x) \Rightarrow Q(x) = \forall x \in \{x : P(x)\}, Q(x)$

$$\begin{aligned} \sim(P(x) \Rightarrow Q(x)) &= \sim(\forall x \in \{x : P(x)\}, Q(x)) \\ &= \exists x \in \{x : P(x)\}, \sim Q(x) \end{aligned}$$

TO DISPROVE $P(x) \Rightarrow Q(x)$, ONE COUNTEREXAMPLE x SUCH THAT $P(x) \wedge \sim Q(x)$ IS TRUE IS SUFFICIENT.

ex.

PROVE OR DISPROVE:

12. If $a, b, c \in \mathbb{N}$ and ab, bc and ac all have the same parity, then a, b and c all have the same parity.

THE STATEMENT IS FALSE. AS A COUNTEREXAMPLE, LET $a = 1, b = c = 2$.

THEN $ab = (1)(2) = 2, bc = (2)(2) = 4$, AND $ac = (1)(2) = 2$ ALL ARE EVEN.

SO ab, bc , AND ac ALL HAVE SAME PARITY, BUT a IS ODD WHILE b & c ARE EVEN.

§9.2 DISPROVING EXISTENCE STATEMENTS

TO DISPROVE $Q: \exists x \in S, P(x)$,
WE MUST PROVE $\sim Q: \sim(\exists x \in S, P(x)) = \forall x \in S, \sim P(x) = x \in S \Rightarrow \sim P(x)$.

COUNTEREXAMPLE NOT SUFFICIENT

CONDITIONAL STATEMENT

- ① DIRECT
- ② CONTRADICTION
- ③ CONTRADICTION

ex.

DISPROVE THE STATEMENT:

THERE EXIST DISTINCT PRIME NUMBERS a & b SUCH THAT $a + b > ab$.

WE DISPROVE THE STATEMENT BY PROVING ITS NEGATION:

IF a, b ARE DISTINCT PRIME NUMBERS THEN $a + b < ab$.

WE DO SO DIRECTLY.

ASSUME a, b ARE PRIME & WLOG ASSUME $a < b$.

THEN $a + b < b + b = 2b \leq ab$, WHERE THE LAST INEQUALITY FOLLOWS FROM THE FACT THAT ALL PRIMES ARE AT LEAST 2.

THUS $a + b < ab$. ■

§9.3 DISPROOF BY CONTRADICTION

To disprove P , ASSUME P IS TRUE & SHOW THIS LEADS TO A CONTRADICTION.

ex. 21. There exist prime numbers p and q for which $p - q = 97$.

We disprove by contradiction.

ASSUME p & q ARE PRIME AND $p - q = 97$, i.e. $p = q + 97$.

(CASE 1) IF $q = 2$ THEN $p = 99 = 9 \cdot 11$ IS NOT PRIME $\Rightarrow \Leftarrow$

(CASE 2) IF $q \neq 2$ THEN q IS ODD BECAUSE 2 IS THE ONLY EVEN PRIME.

SINCE $97 = 2 \cdot 48 + 1$ IS ODD, IT FOLLOWS THAT p IS EVEN,
AS IT IS THE SUM OF TWO ODD NUMBERS.

THEREFORE $p = 2$. THIS IS A CONTRADICTION BECAUSE $p = q + 97$ & q PRIME
IMPLIES $p \geq 99$. $\Rightarrow \Leftarrow$ ■