

Exam 1

Answer all 8 questions for a total of 100 points. Write your solutions in the accompanying blue book. If you solve the problems out of order, please skip pages so that your solutions stay in order. Good luck!

1. Sketch the following set of points in the xy -plane.
 - (a) (6 points) $\{(x, x^2 + y) : x \in \mathbb{R}, y \in \mathbb{Z}\}$
 - (b) (6 points) $\{(x, y) \in \mathbb{N} \times \mathbb{R} : y \geq x\}$
2. Write the following sets in set-builder notation.
 - (a) (6 points) $\{\dots, \frac{5}{8}, \frac{5}{4}, \frac{5}{2}, 5, 10, 20, 40, 80, \dots\}$
 - (b) (6 points) $\left\{\frac{2}{7}, \frac{4}{11}, \frac{6}{15}, \frac{8}{19}, \dots\right\}$
 - (c) (6 points) $\{\{3\}, \{3, 6\}, \{3, 6, 9\}, \{3, 6, 9, 12\}, \{3, 6, 9, 12, 15\}, \dots\}$
3. Draw Venn diagrams for each of the following sets.
 - (a) (6 points) $(A \cap B) - C$
 - (b) (6 points) $(A - B) \cup C$
4. Write each of the following sets by listing its elements between braces or describing it with a familiar symbol or symbols (e.g. interval notation, set-builder notation, \emptyset , \mathbb{Z} , etc.)
 - (a) (6 points) $\bigcup_{n \in \mathbb{N}} \left\{x \in \mathbb{R} : \frac{n-1}{n} < x < \frac{n+1}{n}\right\}$
 - (b) (6 points) $\bigcap_{n \in \mathbb{N}} \left\{x \in \mathbb{R} : \frac{n-1}{n} < x < \frac{n+1}{n}\right\}$
 - (c) (6 points) $\{X \subseteq \{a, b, c, d\} : |\mathcal{P}(X)| = 8\}$
5. (6 points) Are the statements $P \Rightarrow Q$ and $(\sim P) \rightarrow (Q \wedge \sim Q)$ logically equivalent? Support your answer with a truth table.
6. (6 points) Without changing its meaning, convert the following sentence into a sentence having the form “If P then Q ”: An integer is divisible by 8 only if it is divisible by 4.
7. Consider the following mathematical statement.
$$\forall n \in \mathbb{Z}, \exists X \subseteq \mathbb{N}, |X| = n$$
 - (a) (6 points) Rewrite the mathematical statement as an English sentence.
 - (b) (4 points) Is the mathematical statement true or false?
 - (c) (6 points) Write the negation of the mathematical statement in symbols. Then rewrite the negation of the mathematical statement as an English sentence.
8. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
 - (a) (6 points) How many 6-element subsets of A contain exactly 2 even integers?
 - (b) (6 points) How many 6-element subsets of A contain exactly 2 even integers or contain both 8 and 9?