Exam 1 Review

Exam 1 is Friday 2/17 and will cover chapters 1-3 in Book of Proof. The following questions are meant to provide an additional opportunity to practice this material. Solutions are posted to our class website.

1. List the elements of the following sets.
   (a) $\{X \in \mathcal{P}(\{1,2,3\}) : X \in \mathcal{P}(\{1,2\})\}$.
   (b) $\{X \in \mathcal{P}(\{1,2,3\}) : X \subseteq \mathcal{P}(\{1,2\})\}$.
2. Suppose $A = \{0,1\}$ and $B = \{1,2\}$. Find:
   (a) $(A \cap B) \times A$.
   (b) $\mathcal{P}(A) - \mathcal{P}(B)$.
   (c) $\mathcal{P}(A) \cap \mathcal{P}(B)$.
   (d) $\mathcal{P}(A \cap B)$.
3. Prove using logic that for any three sets A, B and C the following identities hold:
   (a) $A - (B \cup C) = (A - B) - C$.
   (b) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.
4. Translate the following sentences into symbolic logic.
   (a) If this gas either has an unpleasant smell or it is not explosive, then it is not hydrogen.
   (b) Having both a fever and a headache is a sufficient condition for George to go to the doctor.
   (c) Both having a fever and having a headache are sufficient conditions for George to go to the doctor.
   (d) If $x \neq 2$, then a necessary condition for x to be prime is that x is odd.
5. Translate these sentences into symbolic logic, then negate them.
   (a) The number x is positive, but the number y is not positive.
   (b) If x is prime, then $\sqrt{x}$ is not a rational number.
   (c) For every prime number p, there is another prime number q with $q > p$.
   (d) If $\sin(x) < 0$, then it is not true than $0 \leq x \leq \pi$.
6. Analyze the logical form of the following statements, then negate them. You may use the symbols $\in, \notin, =, \neq, \wedge, \vee, \Rightarrow, \Leftrightarrow, \forall$ and $\exists$ in your answers, but not $\subseteq, \nsubseteq, \mathcal{P}, \cap, \cup$ or $\sim$ (thus, you must write out the definitions of some set theory notation, and you must use equivalences to get rid of $\sim$).
   (a) $A \subseteq B$.
   (b) $X \in \mathcal{P}(A)$.
   (c) $X \subseteq \mathcal{P}(A)$.
   (d) $X \in \mathcal{P}(A) \cap \mathcal{P}(B)$.
   (e) $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.
7. Let $A = \{n \in \mathbb{N} : n < 10\}$.
   (a) How many disctinct subsets of A are there?
   (b) How many distinct subsets of A with 5 elements are there?
   (c) How many distinct lists with 3 elements taken from A are there?
   (d) How many 8-digit binary strings end in 1 or have exactly four 1's?
   (e) Show that if six integers are chosen at random, then at least two of them will have the same remainder when divided by 5.

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