Exam 2 Review Problems

Exam 2 is Tuesday 4/4 and will cover chapters 4-10 in Book of Proof. The following questions are meant to provide an additional opportunity to practice this material.

Prove the following statements. Use complete sentences.

1. Suppose a, b, c, d are positive integers. If $a \mid b$ and $c \mid d$ then $ac \mid bd$.
2. Suppose $a, b, c \in \mathbb{Z}$, and $n \in \mathbb{N}$. If $a \equiv b \pmod{n}$, and $a \equiv c \pmod{n}$, then $2a \equiv b + c \pmod{n}$.
3. Suppose A and B are sets. Then $A - (A - B) = A \cap B$.
4. The number $\log_2 3$ is irrational.
   Hint: Use proof by contradiction and the fact that $\log_2 3 > 0$.
5. The number $\sqrt{6}$ is irrational.
6. There exists a set X such that $X \cap \mathcal{P}(X)$ is not empty.
   Hint: What element(s) is in $\mathcal{P}(X)$ *no matter what* X *is?*
7. Suppose n is an integer. If $3 \nmid n$, then $3 \mid (n^2 - 1)$.
   Hint: Divide into cases.
8. For all integers $n \geq 1$,
$$3 + 3^2 + 3^3 + \ldots + 3^n = \sum_{i=1}^{n} 3^i = \frac{3^{n+1} - 3}{2}.$$
9. Suppose $x, y \in \mathbb{R}$. If
$$xy - x^2 + x^3 \geq x^2 y^3 + 4,$$
then $x \geq 0$ or $y \leq 0$.
   Hint: Try proving the contrapositive statement.

adamski@fordham.edu