Exam 2

Answer all 7 questions for a total of 100 points. Write your solutions in the accompanying blue book using complete sentences. If you solve the problems out of order, please skip pages so that your solutions stay in order. Good luck!

1. (15 points) Suppose $a, b, c \in \mathbb{Z}$ and $n \in \mathbb{N}$. Prove the statement:
   If $a \equiv b \pmod{n}$ and $a \equiv c \pmod{n}$, then $c \equiv b \pmod{n}$.

2. (15 points) Suppose A, B, and C are sets. Prove the statement:
   If $B \subseteq C$, then $A \times B \subseteq A \times C$.

3. (15 points) Suppose $x, y \in \mathbb{R}$. Prove the statement:
   $(x+y)^2 = x^2 + y^2$ if and only if $x = 0$ or $y = 0$.

4. (15 points) Suppose $a, b \in \mathbb{Z}$. Prove the following statement:
   If the sum $a + b$ and the product ab are both even, then a and b are both even.
   Hint: it is probably easiest to prove the contrapositive statement by cases.

5. (15 points) Prove the statement:
   $\sqrt[3]{2}$ is irrational.

6. (15 points) Prove the statement:
   For every positive integer n,
   $$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}.$$

7. (10 points) *Disprove* the statement:
   For all sets A and B, $\mathscr{P}(A) - \mathscr{P}(B) \subseteq \mathscr{P}(A - B)$.