Homework 2:
1.5 #2, 4, 8
1.6 #2, 6
1.7 #4, 5, 6
1.8 #2, 4, 6, 8
2.1 #4, 6, 8, 10
2.2 #2, 4, 8, 10
2.3 #2, 4, 6, 10

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Exercises for Section 1.5

2. Suppose $A = \{0,2,4,6,8\}$, $B = \{1,3,5,7\}$ and $C = \{2,8,4\}$. Find:

(a) $A \cup B$ **(d)** $A - C$ **(g)** $B \cap C$
(b) $A \cap B$ **(e)** $B - A$ **(h)** $C - A$
(c) $A - B$ **(f)** $A \cap C$ **(i)** $C - B$

(a) $\{0,1,2,3,4,5,6,7,8\}$

or $\{x \in \mathbb{Z} : 0 \le x \le 8\}$

(b) $\emptyset$

(c) A

(d) $\{0, 6\}$

(e) B

(f) C

(g) $\emptyset$

(h) $\emptyset$

(i) C

4. Suppose $A = \{b,c,d\}$ and $B = \{a,b\}$. Find:

(a) $(A \times B) \cap (B \times B)$ **(d)** $(A \cap B) \times A$ **(g)** $\mathscr{P}(A) - \mathscr{P}(B)$
(b) $(A \times B) \cup (B \times B)$ **(e)** $(A \times B) \cap B$ **(h)** $\mathscr{P}(A \cap B)$
(c) $(A \times B) - (B \times B)$ **(f)** $\mathscr{P}(A) \cap \mathscr{P}(B)$ **(i)** $\mathscr{P}(A) \times \mathscr{P}(B)$

(a) $A \times B = \{(b,a),(b,b),(c,a),(c,b),(d,a),(d,b)\}$

$B \times B = \{(a,a),(a,b),(b,a),(b,b)\}$

$\therefore (A \times B) \cap (B \times B) = \{(b,a),(b,b)\}$

(NOTE: THIS IS $(A \cap B) \times B$)

(b) $\{(a,a), (a,b), (b,a), (b,b), (c,a), (c,b), (d,a), (d,b)\}$

(c) $\{(c,a), (c,b), (d,a), (d,b)\}$

(d) $(A \cap B) \times A = \{b\} \times \{b,c,d\}$

$$= \{(b,b), (b,c), (b,d)\}$$

(e) $\emptyset$ (B does not contain any ordered pairs)

(f) The set of all subsets of A that are also subsets of B

$\{\emptyset, \{b\}\}$

(g) Subsets of A that are not subsets of B.

$\{\{c\}, \{d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{b,c,d\}\}$

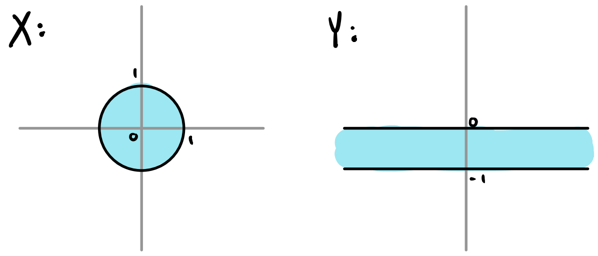
(h) $\mathcal{P}(A \cap B) = \mathcal{P}(\{b\}) = \{\emptyset, \{b\}\}$

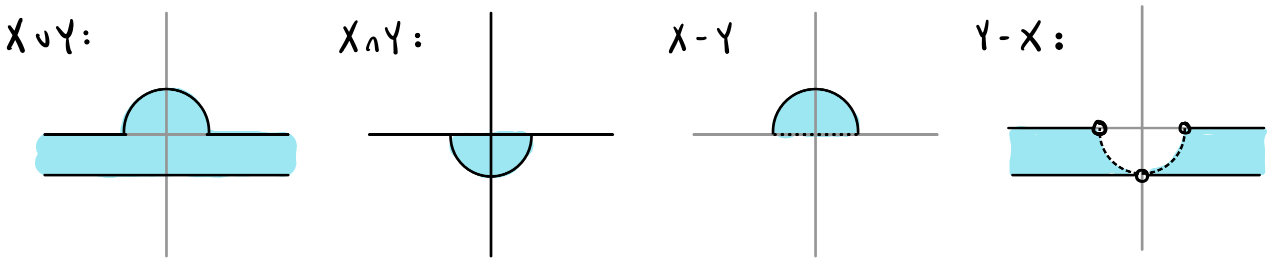
(i) $\mathcal{P}(A) = \{\emptyset, \{b\}, \{c\}, \{d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{b,c,d\}\}$

$\mathcal{P}(B) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

$$\begin{array}{c|cccc}
 & \emptyset & \{a\} & \{b\} & \{a,b\} \\
\hline
\emptyset & (\emptyset,\emptyset), & (\emptyset,\{a\}), & (\emptyset,\{b\}), & (\emptyset,\{a,b\}), \\
\{b\} & (\{b\},\emptyset), & (\{b\},\{a\}), & (\{b\},\{b\}), & (\{b\},\{a,b\}), \\
\{c\} & (\{c\},\emptyset), & (\{c\},\{a\}), & (\{c\},\{b\}), & (\{c\},\{a,b\}), \\
\{d\} & (\{d\},\emptyset), & (\{d\},\{a\}), & (\{d\},\{b\}), & (\{d\},\{a,b\}), \\
\{b,c\} & (\{b,c\},\emptyset), & (\{b,c\},\{a\}), & (\{b,c\},\{b\}), & (\{b,c\},\{a,b\}), \\
\{b,d\} & (\{b,d\},\emptyset), & (\{b,d\},\{a\}), & (\{b,d\},\{b\}), & (\{b,d\},\{a,b\}), \\
\{c,d\} & (\{c,d\},\emptyset), & (\{c,d\},\{a\}), & (\{c,d\},\{b\}), & (\{c,d\},\{a,b\}), \\
\{b,c,d\} & (\{b,c,d\},\emptyset), & (\{b,c,d\},\{a\}), & (\{b,c,d\},\{b\}), & (\{b,c,d\},\{a,b\})
\end{array}$$

8. Sketch the sets $X = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ and $Y = \{(x,y) \in \mathbb{R}^2 : -1 \leq y \leq 0\}$ on $\mathbb{R}^2$. On separate drawings, shade in the sets $X \cup Y$, $X \cap Y$, $X - Y$ and $Y - X$.





NOTE: SOLID BLACK LINES ARE INCLUDED THE SET.
DOTTED BLACK LINES AND OPEN CIRCLES ARE NOT INCLUDED IN THE SET.

Exercises for Section 1.6

2. Let $A = \{0,2,4,6,8\}$ and $B = \{1,3,5,7\}$ have universal set $U = \{0,1,2,\ldots,8\}$. Find:

(a) $\overline{A}$ **(b)** $\overline{B}$ **(c)** $A \cap \overline{A}$ **(d)** $A \cup \overline{A}$ **(e)** $A - \overline{A}$ **(f)** $\overline{A \cup B}$ **(g)** $\overline{A} \cap \overline{B}$ **(h)** $\overline{A \cap B}$ **(i)** $\overline{A} \times B$

(a) $\overline{A} = \{1,3,5,7\} = B$

(b) $\overline{B} = \{0,2,4,6,8\} = A$

(c) $A \cap \overline{A} = \emptyset$

(d) $A \cup \overline{A} = \mathcal{U}$

(e) $A - \overline{A} = A$

(f) $\overline{A \cup B} = \overline{\mathcal{U}} = \emptyset$

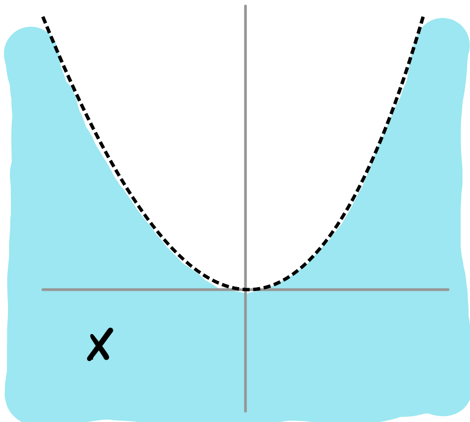
(g) $\overline{A} \cap \overline{B} = B \cap \overline{B} = \emptyset$

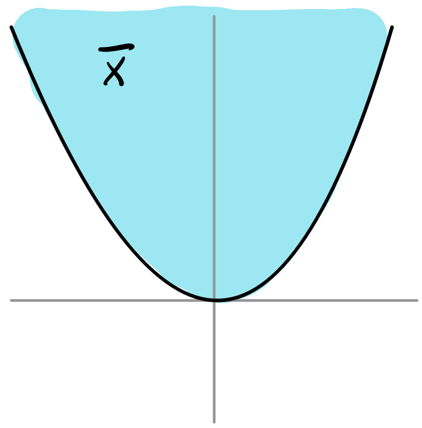
(h) $\overline{A \cap B} = \overline{\emptyset} = \mathcal{U}$

(i) $\overline{A} \times B = B \times B$

$= \{(1,1),(1,3),(1,5),(1,7),(3,1),(3,3),(3,5),(3,7),(5,1),(5,3),(5,5),(5,7),(7,1),(7,3),(7,5),(7,7)\}$

6. Sketch the set $X = \{(x, y) \in \mathbb{R}^2 : y < x^2\}$ on $\mathbb{R}^2$. Shade in the set $\overline{X}$.

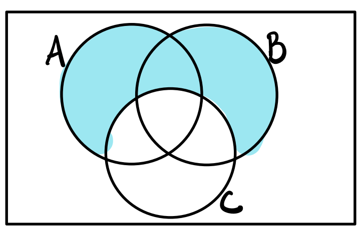




NOTE THAT X DOES NOT INCLUDE ITS BOUNDARY $y = x^2$, BUT $\overline{X}$ DOES.

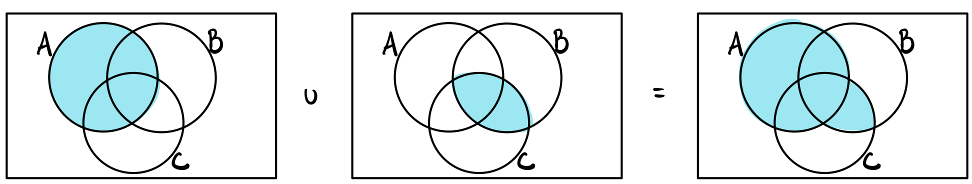
Exercises for Section 1.7

4. Draw a Venn diagram for $(A \cup B) - C$.

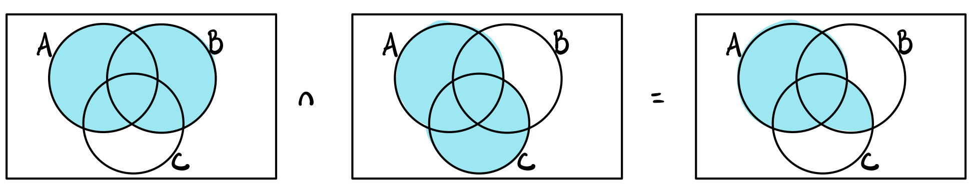


5. Draw Venn diagrams for $A \cup (B \cap C)$ and $(A \cup B) \cap (A \cup C)$. Based on your drawings, do you think $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$?

$A \cup (B \cap C) =$



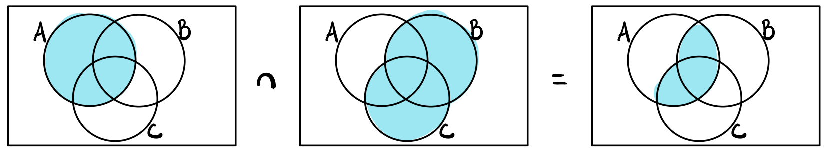
$(A \cup B) \cap (A \cup C) =$



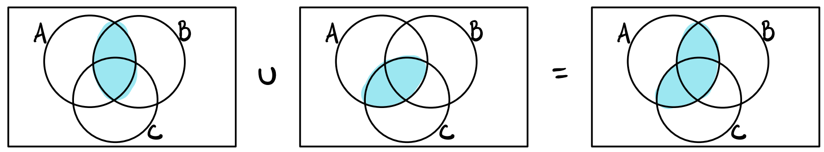
YES, THE 2 SETS ARE THE SAME ✓

6. Draw Venn diagrams for $A \cap (B \cup C)$ and $(A \cap B) \cup (A \cap C)$. Based on your drawings, do you think $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$?

$A \cap (B \cup C) =$



$(A \cap B) \cup (A \cap C) =$



YES, THE 2 SETS ARE THE SAME ✓

NOTE: EXERCISES 5 & 6 PROVIDE A DISTRIBUTIVE RULE FOR HOW THE SET OPERATIONS $\cup$ & $\cap$ INTERACT.

Exercises for Section 1.8

2. Suppose $\begin{cases} A_1 = \{0,2,4,8,10,12,14,16,18,20,22,24\}, \\ A_2 = \{0,3,6,9,12,15,18,21,24\}, \\ A_3 = \{0,4,8,12,16,20,24\}. \end{cases}$

(a) $\bigcup_{i=1}^{3} A_i =$ **(b)** $\bigcap_{i=1}^{3} A_i =$

(a) $A_1 \cup A_2 \cup A_3 = \{0,2,3,4,6,8,9,10,12,14,15,16,18,20,21,22,24\}$

$= \{ x \in \mathbb{Z} : 0 \le x \le 24$ & x IS MULTIPLE OF 2 OR 3 $\}$

(b) $A_1 \cap A_2 \cap A_3 = \{0, 12, 24\}$

4. For each $n \in \mathbb{N}$, let $A_n = \{-2n, 0, 2n\}$.

(a) $\bigcup_{i \in \mathbb{N}} A_i =$

(b) $\bigcap_{i \in \mathbb{N}} A_i =$

(a) $A_1 \cup A_2 \cup A_3 \cup \ldots = \{-2,0,2\} \cup \{-4,0,4\} \cup \{-6,0,6\} \cup \ldots$

$= \{2n : n \in \mathbb{Z}\} = \{\ldots -4,-2,0,2,4,\ldots\}$

(b) $A_1 \cap A_2 \cap A_3 \ldots = \{-1,0,1\} \cap \{-2,0,2\} \cap \{-3,0,3\} \ldots$

$= \{0\}$

6. **(a)** $\bigcup_{i \in \mathbb{N}} [0, i+1] =$

(b) $\bigcap_{i \in \mathbb{N}} [0, i+1] =$

(a) $[0,2] \cup [0,3] \cup [0,4] \cup \ldots = [0,\infty)$

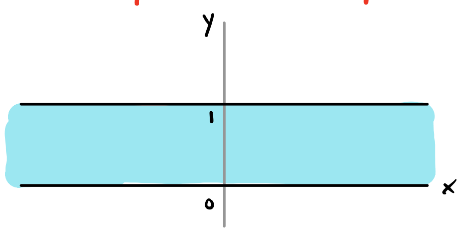
(b) $[0,2] \cap [0,3] \cap [0,4] \cap \ldots = [0,2]$

8. **(a)** $\bigcup_{\alpha \in \mathbb{R}} \{\alpha\} \times [0,1] =$

(b) $\bigcap_{\alpha \in \mathbb{R}} \{\alpha\} \times [0,1] =$

(a) THIS IS THE UNION OF ALL VERTICAL LINE SEGMENTS FROM $(\alpha, 0)$ TO $(\alpha, 1)$, $\alpha \in \mathbb{R}$.

$\{(x,y) \in \mathbb{R}^2 : 0 \le y \le 1\}$



(b) THIS IS THE OF ALL POINTS THAT LIE ON EVERY VERTICAL LINE SEGMENTS FROM $(\alpha, 0)$ TO $(\alpha, 1)$, $\alpha \in \mathbb{R}$.

$\emptyset$

Exercises for Section 2.1

Decide whether or not the following are statements. In the case of a statement, say if it is true or false, if possible.

4. Sets $\mathbb{Z}$ and $\mathbb{N}$. NOT A STATEMENT (NOT EVEN A SENTENCE!)

6. Some sets are finite. STATEMENT, TRUE

8. $\mathbb{N} \notin \mathscr{P}(\mathbb{N})$. STATEMENT, FALSE

FOR ANY SET A, WE HAVE $A \subseteq A$.

THUS $A \in \{X : X \subseteq A\} = \mathscr{P}(A)$

10. $(\mathbb{R} \times \mathbb{N}) \cap (\mathbb{N} \times \mathbb{R}) = \mathbb{N} \times \mathbb{N}$. STATEMENT, TRUE

11. The integer x is a multiple of 7. OPEN SENTENCE

TRUE/FALSE DEPENDS ON VALUE OF X

13. Either x is a multiple of 7, or it is not. STATEMENT, TRUE

TRUE/FALSE DOES <u>NOT</u> DEPEND ON X.

Exercises for Section 2.2

Express each statement or open sentence in a symbolic form such as $P \wedge Q$, $P \vee Q$, $P \vee \sim Q$ or $\sim P$, etc. Be sure to also state exactly what statements P and Q stand for.

2. The matrix A is not invertible.

$\sim P$

WHERE P: THE MATRIX A IS INVERTIBLE

4. $x < y$

$\sim \underbrace{(x \geq y)}_{\sim P}$ OR $\underbrace{(x \leq y)}_{Q} \wedge \sim \underbrace{(x = y)}_{\sim R}$

8. At least one of the numbers x and y equals 0.

$$\underbrace{(x=0)}_{P} \vee \underbrace{(y=0)}_{Q}$$

10. $x \in A \cup B$

$$\underbrace{(x \in A)}_{P} \vee \underbrace{(x \in B)}_{Q}$$

Exercises for Section 2.3

Without changing their meanings, convert each of the following sentences into a sentence having the form "*If P, then Q.*"

2. For a function to be continuous, it is sufficient that it is differentiable.

IF A FUNCTION IS DIFFERENTIABLE
THEN THE FUNCTION IS CONTINUOUS.

4. A function is rational if it is a polynomial.

IF A FUNCTION IS A POLYNOMIAL
THEN THE FUNCTION IS RATIONAL

6. Whenever a surface has only one side, it is non-orientable.

IF A SURFACE HAS ONLY ONE SIDE,
THEN IT IS NON-ORIENTABLE.

10. The discriminant is negative only if the quadratic equation has no real solutions.

IF THE DISCRIMINANT IS NEGATIVE
THEN THE QUADRATIC EQUATION HAS NO REAL SOLUTION.