

Homework 3:

2.4 #2, 4

2.5 #8, 10

2.6 #2, 8

2.7 #2, 6

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Exercises for Section 2.4

Without changing their meanings, convert each of the following sentences into a sentence having the form “ P if and only if Q .”

2. If a function has a constant derivative then it is linear, and conversely.

A FUNCTION HAS A CONSTANT DERIVATIVE IF & ONLY IF IT IS LINEAR.

4. If $a \in \mathbb{Q}$ then $5a \in \mathbb{Q}$, and if $5a \in \mathbb{Q}$ then $a \in \mathbb{Q}$.

$a \in \mathbb{Q}$ IF & ONLY IF $5a \in \mathbb{Q}$.

Exercises for Section 2.5

Write a truth table for the logical statements in problems 1–9:

2. $(Q \vee R) \Leftrightarrow (R \wedge Q)$

		OR	AND	IF & ONLY IF
Q	R	$Q \vee R$	$R \wedge Q$	$(Q \vee R) \Leftrightarrow (R \wedge Q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

4. $\sim(P \vee Q) \vee (\sim P)$

P	Q	$P \vee Q$	$\sim(P \vee Q)$	$\sim P$	$\sim(P \vee Q) \vee (\sim P)$
T	T	T	F	F	F
T	F	T	F	F	F
F	T	T	F	T	T
F	F	F	T	T	T

8. $P \vee (Q \wedge \sim R)$

P	Q	R	$\sim R$	$Q \wedge \sim R$	$P \vee (Q \wedge \sim R)$
T	T	T	F	F	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	F

10. Suppose the statement $((P \wedge Q) \vee R) \Rightarrow (R \vee S)$ is false. Find the truth values of P, Q, R and S. (This can be done without a truth table.)

THE ONLY WAY FOR $((P \wedge Q) \vee R) \Rightarrow (R \vee S)$ TO BE FALSE IS

FOR $((P \wedge Q) \vee R)$ TO BE TRUE AND $(R \vee S)$ TO BE FALSE

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

① So BOTH R & S ARE FALSE

② Now we know R is FALSE

③ So $P \wedge Q$ MUST BE TRUE!
THAT IS, BOTH P & Q ARE TRUE.

IN SUMMARY,
 P : TRUE
 Q : TRUE
 R : FALSE
 S : FALSE

Exercises for Section 2.6

A. Use truth tables to show that the following statements are logically equivalent.

2. $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$

P	Q	R	$Q \wedge R$	$P \vee Q$	$P \vee R$	$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

✓

8. $\sim P \Leftrightarrow Q = (P \Rightarrow \sim Q) \wedge (\sim Q \Rightarrow P)$

P	Q	$\sim P$	$\sim Q$	$P \Rightarrow \sim Q$	$\sim Q \Rightarrow P$	$\sim P \Leftrightarrow Q$	$(P \Rightarrow \sim Q) \wedge (\sim Q \Rightarrow P)$
T	T	F	F	F	T	F	F
T	F	F	T	T	T	T	T
F	T	T	F	T	T	T	T
F	F	T	T	T	F	F	F

✓

Exercises for Section 2.7

Write the following as English sentences. Say whether they are true or false.

2. $\forall x \in \mathbb{R}, \exists n \in \mathbb{N}, x^n \geq 0$ For every real number x , there is a natural number n such that $x^n \geq 0$.

True.

6. $\exists n \in \mathbb{N}, \forall X \in \mathcal{P}(\mathbb{N}), |X| < n$ There exists a natural number n such that every subset of the natural numbers has less than n elements.

False.