EXAM 2 STUDY GUIDE

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Sets

You need to understand what the following sets are.

(1) $\emptyset = \{\}$ (emptry set)
(2) $\mathbb{N} = \{1, 2, 3, \ldots\}$ (natural numbers)
(3) $\mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$ (integers)
(4) $\mathbb{Q} = \left\{ \frac{a}{b} \;:\; a, b \in \mathbb{Z}, b \neq 0 \right\}$ (rational numbers)
(5) $\mathbb{R}$ (real numbers)
(6) Intervals on $\mathbb{R}$:
    (a) $(a, b) = \{x \in \mathbb{R} \;:\; a < x < b\}$
    (b) $[a, b] = \{x \in \mathbb{R} \;:\; a \leq x \leq b\}$
    (c) $[a, \infty) = \{x \in \mathbb{R} \;:\; a \leq x\}$
    (d) etc.

Definitions

You should know the following definitions and know how to use them.

(1) An integer n is *even* if $n = 2a$ for some $a \in \mathbb{Z}$.
(2) An integer n is *odd* if $n = 2a + 1$ for some $a \in \mathbb{Z}$.
(3) A real number r is *rational* if $r = a/b$ for some $a, b \in \mathbb{Z}$; it is *irrational* if there are no integers $a, b \in \mathbb{Z}$ with $r = a/b$.
(4) Suppose $a, b \in \mathbb{Z}$. We write $a \mid b$ and say *a divivides b* if $b = ka$ for some $k \in \mathbb{Z}$.
(5) Suppose $a, b, n \in \mathbb{Z}$ and $n \geq 2$. We write $a \equiv b \pmod{n}$ and say *a is equivalent to b modulo (mod) n* if $n \mid (a - b)$.
(6) The *Cartesian product of A and B* is $A \times B = \{(x, y) \;:\; x \in A, y \in B\}$.
(7) The *power set of A* is $\mathcal{P}(A) = \{X \;:\; X \subseteq A\}$
(8) We write $A \subseteq B$ and say *A is a subset of B* if $(x \in A) \Rightarrow (x \in B)$.
(9) We write $A = B$ and say *A equals B* if $A \subseteq B$ and $B \subseteq A$.

Techniques/Concepts

(1) Direct proof
(2) Contrapositive proof
(3) Proof by contradiction
(4) If-and-only-if proof
(5) Effective use of cases
(6) Proof of existence statements
(7) Mathematical induction (strong)
(8) Disproof by counterexample (smallest)
(9) De Morgan's laws
    (a) $\sim (P \vee Q) = (\sim P) \wedge (\sim Q)$
    (b) $\sim (P \wedge Q) = (\sim P) \vee (\sim Q)$